Towards Location Specific Statistical Fracture Prediction in High Pressure Die Castings

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1 Abstract

High pressure die casting is an economical means of producing a high volume of aluminium parts, with a design freedom that can enable lighter structures to be envisioned, compared with wrought assemblies. However, cast aluminium parts have been shown to be vulnerable to damage by defects caused by the entrainment of air during the casting process.
A recently developed entrainment prediction algorithm, which is believed to more quantitatively predict the distribution of entrainment defects within a casting, was used to predict the distribution of these defects for two variants of the casting process for a commercial part. Using a novel fuzzy statistical correlation method, the predicted distribution of entrainment damage was correlated with the statistical distribution of entrainment damage, as determined by tensile testing.
This work demonstrates a verification of the mapping methodology, in which the correlated strength distribution was mapped into LS-DYNA models of the test bars used for correlation. The results showed that the fuzzy statistical entrainment damage model can be tightly fitted to tensile test data, and that this fidelity can be reproduced in LS-DYNA simulations using the methods described, however further work is required to demonstrate the method's predictive capability.

2 Introduction and Background

2.1 Introduction

High Pressure Die Castings are widely used in the automotive industry, because they may be designed with relatively complex geometries, and produced in high volumes at low cost. However, they are often designed based on a conservative strength estimate, because the factors which influence their strength are difficult to account for. One of these factors is the unique distribution of entrainment defects within each casting.
The aim of this work is to summarise and demonstrate some recent developments and insights, regarding the nature of entrainment defects in cast aluminium parts. When this new understanding is formulated into a set of mathematical models, it can be applied to generate a map of the statistical strength distribution in a cast part. This strength distribution is based on the distribution of entrainment defects predicted by a simulation of the casting process, and can in turn be mapped into an LS-DYNA key-file, to simulate the failure of the part.

2.2 Entrainment Defects

Since the first edition of Campbell's "*Castings*"[1], awareness has been growing within the aluminium industry of defects known as oxide bi-films. It is supposed that these defects form when the oxidised surface of the molten aluminium is folded over (dry side to dry side) and taken beneath the surface by fluid flow during casting. As such, the bi-film defect forms a discontinuity in the metal, which may nucleate other forms of defect [2] as the casting solidifies, or simply form a site for crack initiation.
It has been shown [3] that oxide bi-films formed during sand casting experiments can significantly reduce the ultimate tensile strength (UTS) of these castings. Additionally, these defects are more likely to cause significant damage in "turbulent" fluid flow conditions, which supports the assumption that the defects are more likely to be formed under such conditions.
If oxide bi-films are formed by the action of fluid turbulence mixing air with liquid metal, then it might be supposed that High-Pressure Die Castings would be extremely vulnerable to this kind of defect. However, during the fractography of High Pressure Die Cast samples, the author has observed very few defects typical of oxide bi-films. Instead, related defects were observed, which more closely resemble bubbles. The crinkled appearance of inner surface of these bubbles (Figs. 1 A and B),

together with an oxygen peak on Energy Dispersive X-ray spectroscopy (EDX) (Fig. 1 C) shows that this surface reacted with air in the liquid state, and suggests that the bubble was also formed by the entrainment of air.

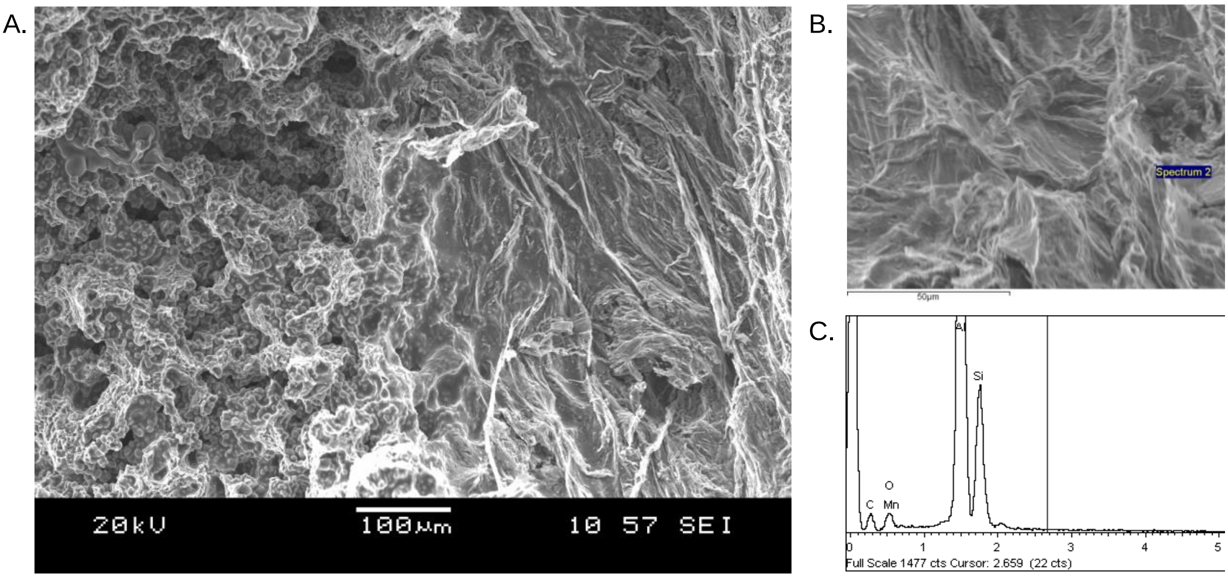

Fig.1: An SEM image of an air-pore (A, right), associated with a region of micro-shrinkage porosity (A, left), alongside with a higher magnification view of the air-pore (B), showing the location that the EDX spectrum (C) was sampled from.

2.3 Failure Statistics of Cast Parts

If a casting is made in a way that consistently encourages the formation of these entrainment defects (i.e. turbulent), there will not be a constant reduction in strength, compared to a similar casting that is made in a way that consistently discourages the formation of such defects (i.e. tranquil).

When considering the influence of defects, the strength of cast parts should be described statistically, rather than assuming a uniformly reduced strength. The probability of failure is often described using the 2-parameter Weibull distribution [4] below:

$$P(X < x) = 1 - \exp\left[-\left(\frac{V}{V_0}\right)\left(\frac{x}{x_0}\right)^m \right] \tag{1}$$

Here, X is used as a random variable, which could represent any fracture parameter, for example ultimate tensile strength or elongation to failure. For those cases, x would represent the applied stress or strain, respectively. As x increases, the more likely it is to exceed X, and the more likely it is that failure would occur at that load.

The scale parameter, x_0, is the load which would cause failure 67.2% of the time, and can be considered like an average for the distribution. The shape parameter, or Weibull modulus, m, describes the spread in the measured strength. The value of m must be positive, and the larger it is, the less deviation there is from the value of x_0.

Also included in eq.1 is the volume scaling term, V/V_0, which is needed to account for the effect of differently sized test coupons, or the strength of sub-volumes, or elements, of a part. The Weibull distribution is an extreme value distribution, as it describes the strength of a component in terms of the *weakest* part of that component, such as the volume of material where stress is concentrated by a defect.

Suppose that V_0 represented the gauge volume of a test-bar. If two test bars were linked together, (V/V_0=2), then the estimated strength of this pair would be lower than the estimated strength of either test-bar on its own. At any given stress the probability of failure for the pair would be given by the probability that *either* the first bar fails *or* the second bar fails.

When this distribution is used to compare test results from castings, it is often found that relatively "poor quality" castings have a lower Weibull modulus, where highly reliable castings have a high Weibull modulus. As such, the Weibull modulus has come to describe the quality of castings.
The Weibull modulus should be an important consideration when designing with castings. Suppose that a part should be designed to have a 5% chance of failure, when subjected to some extreme load case. Assuming an x_0 value of 100 MPa, and a modulus of 30 (indicating a "good" casting), then the allowable stress would be 90.5 MPa. However, with a modulus of 10 (indicating of a "bad" casting), then the allowable stress would be reduced to 74.3 MPa.
Statistical strength variation of this type means that the strength varies not just from part to part, or test-sample to test-sample, but also within each sample. Therefore, the goal must be a mapping where the strength of each element is statistically sampled.

2.4 Statistical Models of Defect Damage

Assuming that the stress-strain curve remains relatively fixed, if the load at failure in a casting varies, this must be understood as the result of a different defect initiating failure in each case.
Nyahumwa [5] investigated the pore size distribution in Al-Si-Mg castings, and found that the log-normal distribution seemed most appropriate. Tirakioğlu [6] reviewed a number of similar investigations, and found strong support for the assumption that defect size follows a log-normal distribution.
A fact that that **Fehler! Verweisquelle konnte nicht gefunden werden.** does not highlight, is that a sample may contain multiple defects. Each of these defects has the potential to initiate failure, but only the most damaging, the "critical defect", will actually initiate failure. As such, the damage distribution of the critical defects must be inferred from the bulk distribution using an extreme value distribution. Just as the Weibull distribution (Eq. 1) describes the minimum strength, a related distribution can describe the most damaging defect.
Fatigue life is very sensitive to the most damaging initial defect, since a fatigue crack will grow faster from a more damaging defect. It is then possible to relate the fatigue life to the initial size of this critical defect by integrating the Paris Law. Tirakioğlu [6] used fatigue life data for Al-Mg alloys from several sources to infer the effective initial size distribution of the critical defects, then fitted the Generalised Extreme Value (GEV) distribution to those data. The GEV distribution is a generalisation of the 3 types of extreme value distribution, Weibull, Gumbel and Fréchet, where the value of one of its parameters, ξ, essentially switches between those distributions. As such, the best-fit value of that parameter can be used to infer which of those 3 types best fit the data.
For the majority of cases, it was found that the Gumbel distribution was appropriate to describe the critical defect size distribution. Tirakioğlu states [6] that this result would be expected, if it is assumed that the bulk population of defects follows a log-normal distribution.
However, for each data set investigated, the 95% confidence intervals presented for ξ indicated that the data could also belong to the a Fréchet distribution, given below in 3-parameter form:

$$P(D < d) = \exp\left[-\left(\frac{d-\mu}{d_0}\right)^{-n}\right] \tag{2}$$

D is used as a measure of the damage capability of the critical defect in a sample, such as its size. As such, d represents the damage required to initiate fracture in the material at some load. As the load on the part increases, a less damaging defect can initiate failure, and so d becomes smaller. When d becomes smaller than D, the most damaging defect in the sample exceeds the damage required for the part to fail, and the part fails.
Similar to the Weibull distribution, n should be positive; the more negative the value of $-n$, the less deviation there is from the value of d_0.
The third parameter, μ, is a threshold or offset, which sets the minimum value for D. If μ is positive, then there will always be some damage in the sample.
One advantage of the Fréchet distribution is its close relation the Weibull distribution, in 2 parameter form. To demonstrate this, let it be temporarily assumed that μ can be treated as 0, a negative power law relation can be derived between x and d, as follows:

$$P(X < x) = 1 - \exp\left[-\left(\frac{x}{x_0}\right)^m\right] \equiv P(Fail) \quad (3)$$

$$P(X > x) = \exp\left[-\left(\frac{x}{x_0}\right)^m\right] \equiv P(Survive) \quad (4)$$

Use (2) in 2-parameter form (μ=0)

$$P(D < d) = \exp\left[-\left(\frac{d}{d_0}\right)^{-n}\right] \equiv P(Survive) \quad (5)$$

Equate (4) and (5) and take logarithms.

$$-\left(\frac{d}{d_0}\right)^{-n} = -\left(\frac{x}{x_0}\right)^m \quad (6)$$

$$\left(\frac{x}{x_0}\right) = \left(\frac{d}{d_0}\right)^{-n/m} \quad (7)$$

$$x = \frac{x_0}{d_0^{-n/m}} d^{-n/m} \quad (8)$$

This is significant, because such negative power law relations also occur in fracture mechanics, such as eq. 9, [Knott], which relates fracture stress, σ_F, and a, which is the half length of an embedded crack.

$$\sigma_F = K_{crit}(\pi a)^{-1/2} \quad (9)$$

The negative power law form would remain equally valid, if substitutions are made for σ_F, or a using a power law. For example, a could be substituted for a characteristic area or volume of the defect. Similarly, if the power law of strain hardening [7], eq. 10, may be used to relate stress and strain, then x can be equated to stress, strain, or some functions of the two.

$$\sigma = C\varepsilon^n \quad (10)$$

This range of applicable approximations would make a 2-parameter Fréchet distribution an appealing model for damage, particularly if the strength is assumed to always follow a 2-parameter Weibull distribution. However, Tirakioğlu [6] showed that the Gumbel distribution did provide a good fit for the linear size of the critical defects, and the 2-parameter Weibull distribution is merely a convenient descriptive model.

The addition of the threshold parameter, μ, adds another level of flexibility; the more negative the value of μ, the more the Fréchet distribution tends towards the Gumbel distribution. If the value of μ is positive, this sets a lower limit on damage, and therefore an upper limit on the strength of each sample.

2.5 Modelling Defect Formation and Transport

Casting simulation software, such as MAGMAsoft and ProCAST, is commonplace in modern foundries. By simulating the flow of fluid and heat in the mould, they provide a good indication about how and where many common types of casting defect might occur. However, the algorithms used to predict and quantify the formation of entrainment defects have still not reached maturity, although it is possible to use these models' outputs to inform design.

Dørum et. al. [8] developed an empirical model based on MAGMAsoft's "air contact time" criterion, which allowed them to produce a statistical strength map for a cast U-section. Importantly, their model was based on stochastic principles, in that strength values were sampled from an interpolated Weibull distribution that was based on tensile data from “good” and “bad” regions of the casting. The team could then use the air contact time output as the basis for interpolation between the statistical tensile properties of the “good” regions of the casting, and the properties of the “bad” regions of the casting. A statistical strength map can thus be created, which can be sampled multiple times to represent multiple castings.

Air contact time provides an indication of the length of time that each volume of fluid has spent in contact with the air, as it travels through the mould, and is one of the simplest models to represent oxide film damage. However, it is not at all robust; an oxide film which forms over 2s will behave very similarly to an oxide film which forms over 0.2s. It also does not directly model entrainment, but relies on scalar advection to carry “oxidised” fluid below the surface. As such, “oxidised” material that is not carried beneath the surface by fluid, would still appear damaged, although it would not have been entrained.

However, for a model to adequately predict the final defect distribution within a casting, it must also be able to predict where the defects will be end up, as well as when and where they are created. This presents a problem for scalar transport models, such as air contact time, since the scalar quantity (e.g. time) is carried from one cell to the next by fluid flow; there is no way to model the dynamics of the defects within the fluid. One way around this is to represent the defects individually, using Lagrangian Particle Tracking (LPT). This technique was originally proposed as a solution to the problem of numerical diffusion, an artefact of Eulerian methods, in which scalar quantities are gradually smoothed out when their value is repeatedly partially transported from one cell to the next [Reilly]. However, using a Lagrangian particle method also allows the defects to be modelled with mass, and for buoyancy and drag forces to be applied. The coupling is only one way, however, meaning that any particles do not affect the fluid, or each other.

Reilly [9] developed an algorithm named the Boolean Oxide Entrainment Code (BOEC), as an attempt to create a practical model to quantitatively assess the severity of entrainment damage in castings. This model was based around FLOW-3D, a highly capable general purpose flow solver, which can be easily modified with user subroutines. This allowed Reilly to define “entrainment” using a set of 48 logical criteria, allowing a subroutine to scan the fluid flow for an entraining flow pattern on every time step. Then, particles may be placed in these areas to represent defects, and tracked to their final location using FLOW-3D’s Lagrangian particle tracking algorithm.

The Boolean Oxide Entrainment Code (BOEC) has been successfully applied on to a number of cases, one of the most informative of these is work by Yue [10], in which test bars were taken from a number rectangular plates, which were cast using 3 different running systems. The flow pattern was simulated with the BOEC for each of the 3 cases, and the modelled plate castings were divided up in the same way that test bars were extracted from the experimental castings. The number of particles predicted to end up in each of these divisions was counted by an in house code, so that the particle distribution could be compared to the strength distribution. A location for location correlation between strength and particle count was not evident, however, when the reciprocal of particle count was plotted on a Weibull chart, some correlation could be drawn in a statistical sense. In particular, it was shown that the plunging jet case was the most unreliable in terms of strength, but also showed a consistently high particle count.

Beyond that, any links were inconclusive, which could be partially attributed to flaws in the BOEC algorithm. The most significant of these flaws is that the since entrainment is defined logically, any given flow pattern is either entraining, or not entraining; there is no measure of the relative severity of entrainment to distinguish between one entraining flow and another, therefore there is no reason to suppose proportionality between particle count and entrainment damage. Additionally, the logical criteria are based on human experience rather than a mathematical model.

In a previous work, Watson [11] demonstrated an algorithm called the Surface Area Entrainment Code, which was also based on FLOW-3D and used a Lagrangian particle representation of defects. The algorithm has improved robustness over previous models, particularly for high velocity or non-intuitive flow patterns. In that paper, a rough location-for-location correlation was found between the damage measured using tensile test statistics and the particle counts at locations corresponding to the test bar gauge lengths. However, only 1 flow case was presented, involving 3 test bar locations.

2.6 Inferring Strength Distribution from Defect Distribution

Returning to Yue’s work, the approach of considering the distribution of particle counts at different locations in a statistical sense offers an interesting contrast to a location-for-location approach.

Consider a hypothetical casting simulation, which is set up exactly right, and produces a map of “damage density”, some function of the number and severity of defects, that corresponds inversely to strength.
If the simulation is perfect, then the defect number density predicted at various locations should exactly correlate with the average strength of test bars sampled from equivalent locations on the real castings. In reality, when this part is cast, chaotic flow variations would arise from turbulence and process variability, and therefore no two parts would ever be identical. Since a “perfect” simulation is repeatable, it does not indicate how the castings may vary.
Ideally, multiple casting simulations might be run, with their boundary conditions changing in line with real-world process variation, so that the variation in damage density at specific locations might be accounted for.
Assuming the process variation is sufficiently small, then similar quantities of defects will be created at similar locations and times, and carried to their final positions by similar fluid flow patterns. If the damage density in each simulated case is then is taken at various points, and plotted statistically as in Yue’s work, then each damage distribution might be very similar, even though no simulation would be the same in a location for location sense.
In the simulation results, it might appear as though parcels of fluid with higher or lower defect concentrations would be randomly displaced a short distance from their average positions. The case to case variation in damage at a specific point should therefore reflect the varying defect concentrations in the nearby fluid parcels which might end up at that point.
This is the basis which will be used to extrapolate the damage variation at any given point, from a single simulation.

3 Method

3.1 Casting and Tensile testing

Test castings based on a commercial High Pressure Die Cast part were produced using two modifications of the part’s normal process. The first set of 15 left and right pairs of castings were cast using the usual mould geometry, but with a reduced shot speed. These will be referred to as the “Full Gate” castings. The second set of 15 pairs were cast with one side of the in-gate blanked off using a piece of aluminium, to strongly modify the flow pattern, and were cast using the same shot speed as the first set. These will be referred to as the “Blanked gate” set.

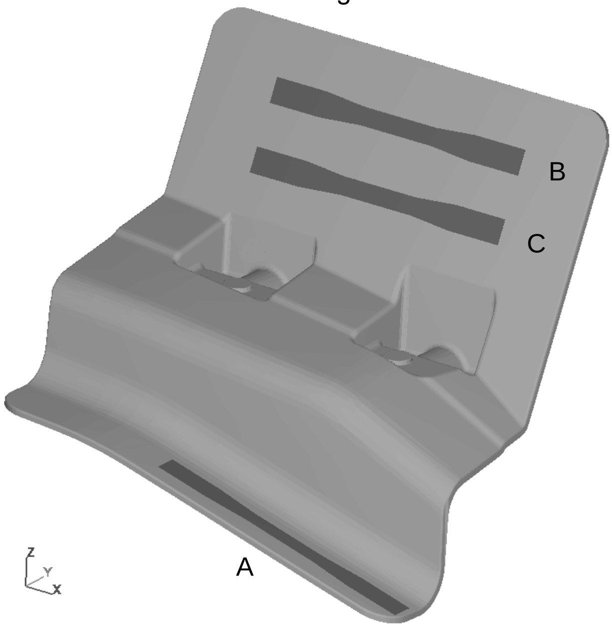

Fig.2: The locations on the trial castings where the test bars were extracted from.

Test bars were extracted from each of these castings as per Figure 2, and tested under uniaxial tension at a strain rate of 3 mm/min.

Tensile test data for each combination of location and casting variant were grouped into a total of 6 sets of 30 tests. The true fracture stresses were calculated, and Maximum Likelihood Estimation (MLE) was used to characterise the data-sets assuming a 2-parameter Weibull distribution.

3.2 Casting Simulation

The two variants of the casting process were simulated using the commercial CFD package, FLOW-3D. The Surface Area Entrainment Code [11] mentioned previously was included in these simulations as a user subroutine. The entrainment algorithm was set up to place one Lagrangian particle into the fluid, at the point of entrainment, for each mm^2 of entrained fluid surface predicted. Particles placed in this way were carried by the simulated fluid until the casting is solidified enough for fluid motion to cease.

3.3 Statistical Mapping and Optimisation

As illustrated in section 2.6, it is possible to infer the both the average damage, and the variation in damage at any given point, under the assumption that the fluid transporting the defects will end up at slightly different locations in each casting. Based on this reasoning, a random walk algorithm was developed, to sample the smoothed defect number density from the casting simulation results in the neighbourhood of a given point, and then fit a 3-parameter Fréchet distribution to the sampled data.
The geometry information from an LS-DYNA model of the test bars was rotated, translated and scaled, so that their position in the simulated castings was equivalent to the test bar sampling locations illustrated in Figure 3. The centroid of each element in the gauge length of each test bar model was used as a starting point for the above random walk algorithm. In this way, the damage in the neighbourhood each element was characterised statistically, for each tensile data set.
In fact, this process was performed with 50 different values for the random walk distance, and the initial smoothing radius, in order to find the values for these parameters, which provide the best final fit.

3.4 Calibration of Statistical Transform

For the mapping function, an inverse power law relationship was assumed between the damage, characterised by the defect number density, and true fracture stress, as below, where B is negative:

$$\sigma_F = A \cdot D^B \tag{10}$$

In addition to these two parameters, two more are required, one which compensates for the discrepancy in sampling volume between the test-bar gauge length and the random walk damage sampling algorithm, and another, which helps to account for defects which were present in the metal before the scope of the simulation, for example during the filling of the shot sleeve.
The fitting algorithm used uses an arithmetic least squares solution to fit A and B to the Weibull statistics estimated for the tensile data sets, whilst numerically iterating the other two parameters to find the best overall fit.

3.5 Damage Mapping and Verification

Once the optimum mapping parameters were found, these were used as inputs to a PRIMER script, which assigns a strength distribution to a set of **`*PART`**s within an LS-DYNA model.
Variation in strength is modelled by first sampling the statistical damage distribution corresponding to each element, and so calculating the strength value for each element. However, instead of assigning an individual strength value to each element, a binning algorithm is used to group elements with similar strengths together into a number of “bins”; in this exercise 50 bins were used for each test bar.
New **`*PART`** ids are then created corresponding to each bin, based on a master **`*PART`**. A unique material id is also created for each new **`*PART`**, so that its strength can be set to the average strength of the elements in each bin using the **`*MAT_ADD_EROSION`** keyword. The elements grouped into each bin are then assigned the **`*PART`** id corresponding to each bin.
Additionally, the script outputs a summary of the binning results, which was used to evaluate how effectively the algorithm reproduced the intended strength statistics.

4 Results

4.1 Fluid Simulation

The predicted geometric distribution of defects for each simulated case is presented in Figure 3. It is evident that blanking part of the gate has significantly affected the fluid flow, by the difference in the final defect distribution between the two cases.

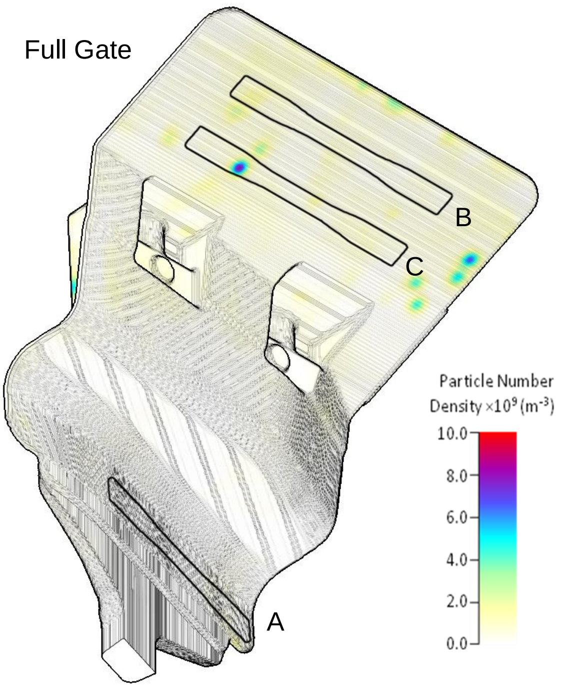

Fig.3: Final time frame of "Full Gate" and "Blanked Gate" simulations, coloured by defect number density, with test bar positions overlaid.

4.2 Damage Mapping and Verification

Figure 4, below, presents some examples of the results of the mapping process, with the **`*PART`**s assigned a colour between red (200 MPa) and yellow (300 MPa) depending on the strength of the material. It can be seen that some regions are of much more uniform strength than others, and also that some regions are of generally higher or lower strength, visually demonstrating the statistical nature of the mapping process.

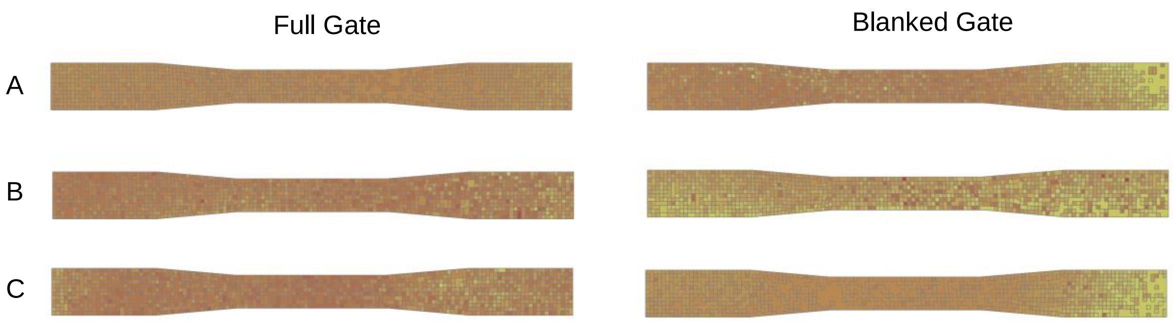

Fig.4: Example strength mappings of each test bar.

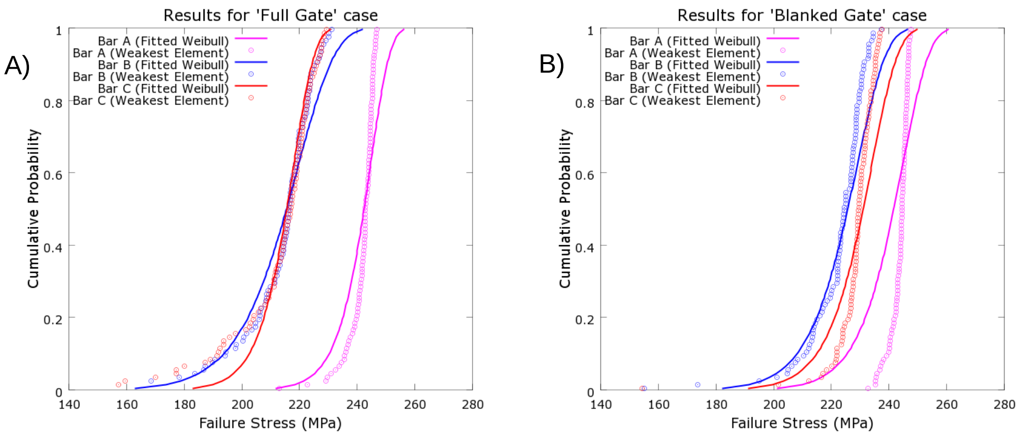

Fig.5: A comparison of the cumulative distributions of true fracture stress produced by repeated evaluation of the defect mapping algorithm, and the Weibull statistics that the mapping parameters are based on.

The mapping process was run 100 times for each combination of casting variant and test bar location, and for each mapping, the strength of the bar was taken as the strength of the weakest element in the gauge length. The resulting strength distributions are presented as a cumulative distribution in Figure 5, alongside the Weibull distributions that the mapping parameters were fitted to. The fit is very close in the mid-section of each curve, however this fidelity is only maintained in the lower tail in the data for both data-sets relating to test bar location B. For the upper tail, only test bar location C in the Full Gate case maintains a close fit.

5 Discussion and Summary

The results presented demonstrate that the defect prediction algorithm used in the fluid simulation can produce a sufficiently realistic defect distribution, that this can be correlated with a statistical damage function with a reasonably close fit, although the variation in each test-bar group is a consistently under-estimated, particularly towards the upper tail of each distribution. If the results where the lower tail is underestimated were used for design, then the part may have insufficient durability.
Additionally, the mapping methodology itself has been shown to produce a believable strength distribution within each mapped test bar- the use of a detailed mapping methodology gives perhaps the best indication of the strength of regions which did not fail in tensile tests.
Figure 6 indicates that there is a distinct upper limit to the mapped strength distribution, as would be expected, since the mapping is based a negative power of a Fréchet type distribution. For most of the present cases, this means that the strength of the strongest test pieces is under-estimated. However, an advantage of this feature is that as elements are scaled to smaller volumes, their strengths do not become unrealistically high.
Another potential advantage of the methodology presented here is that the process of mapping the statistical data into an LS-DYNA simulation is done using a PRIMER script, which is relatively easy to implement in a commercial setting.

6 Literature

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