Investigation of Failure Criterion in Dynamic Torsion Tests with Solid Cylindrical Specimens

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Abstract

When investigating the limiting states of materials under dynamic loading conditions, it's important to specify the dependency of plastic failure strain on the stress state. Usually, such dependence is build upon the experimental data obtained from dynamic tests in tension and compression of solid cylindrical specimens with different working part geometry, followed by a monotonic extrapolation. In the recent studies [1] the existence of complex, non-monotonic dependence of failure strain on the stress state parameters is shown for a number of materials. In these cases, a mentioned set of tests is not enough to construct a reliable criterion relations. In statics, one of the most informative experiments for the failure criterion construction is a torsion test on solid or thick-walled cylindrical specimens. Although a nonuniform stress state arises in the sample in this case, effective methods of its interpretation are developed [2,3]. The theory of this experiment conformably to the dynamic processes at large plastic strains has not yet been developed. Using the LS-DYNA implemented virtual test bench, the experimental setup for the solid cylinder torsion test with high strain rates and methods of its stress state identification are discussed. It is shown that for the strain rate range of 10^2*-*10^4 *1/s the kinematic hypotheses that are taken in the quasi-static torsion are valid, that allows the effective use of known methods of the sample's stress state decoding.*

Keywords: *Plasticity, failure, large strains, experiment, torsion, modeling, imaginary tube.*

Introduction

Adequate construction of the failure criterion involves an experimental study of fracture in different types of stress state, while experiments in tension and compression traditionally used for this purpose are not enough by force of a significant nonmonotonicity of fracture surface. For the construction of the fracture surface in this case, besides compression and tension experiments, additional shear tests as well as combined biaxial tests with thin-walled tubular specimens of different geometry should be conducted. Being fully justified to determine the dynamic deformation curves, this approach may prove to be insufficiently substantiated in the investigation of fracture due to loss of stability of such samples before failure, and, accordingly, the emergence of significant heterogeneity of the stress state in the specimen. In statics, one of the most informative and reliable experiments to construct the fracture surface is a test on the pure torsion or torsion combined with tension (compression) using solid or thick-walled cylindrical specimens (also the internal and external pressures can be allowed). Despite the inhomogeneous state is realized in such specimens, effective methods for its interpretation are developed [2]. This approach was also extended to large strains [3]. In general triaxial case, these methods are the generalizations of imaginary tube and a degenerate imaginary tube methods [2], which are based on the natural kinematic hypotheses of the radial fiber straightness and the cross section rigid body rotation, together with the incompressibility condition.

Degenerate imaginary tube method

Consider, for brevity, the method of degenerate imaginary tube with respect to biaxial loading of a thick-walled cylindrical specimen with length l, inner radius r_1 and outer radius r_2 in the initial configuration by axial force P and torque M. In this case, in accordance with the above mentioned hypothesis, the relationship between the specimen coordinates in the actual and initial configurations relative to the natural cylindrical coordinate system has the form:

$$R(t) = (l / L(t))^{1/2} r$$
$$\Phi(t) = \phi + (\Theta(t) / l) z$$
$$Z(t) = (L(t) / l) z$$

It's convenient to use the stress tensor $\Sigma^R = (\sigma_{ij})$ and the strain $\mathrm{E}^R = (e_{ij})$ tensor to represent the stress-strain state in the sample in this case [2], which are defined by the Cauchy stress tensor Σ and the strain rate tensor D components in a moving orthonormal frame $\vec{k}_i$, bound to the coordinate vectors of natural cylindrical basis, so that

$$\mathrm{E}^R = \int_0^t R^t D R dt, \quad \Sigma^R = R^t \Sigma R$$

$$\left\{ \begin{array}{l} \dfrac{d\vec{k}_i}{dt} = R\vec{k}_i \\ \vec{k}_i \Big|_{t=0} = \vec{e}_i \end{array} \right\}, \quad \left\{ \begin{array}{c} \vec{k}_1 \\ \vec{k}_2 \\ \vec{k}_3 \end{array} \right\} = \left\{ \begin{array}{c} \vec{e}_R \\ \vec{e}_\Phi \\ \vec{e}_Z \end{array} \right\}, \quad \left\{ \begin{array}{c} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{array} \right\} = \left\{ \begin{array}{c} \vec{e}_r \\ \vec{e}_\phi \\ \vec{e}_z \end{array} \right\}$$

In this case, the E^R strain tensor components inside the sample can be expressed through local axial ε and shear γ deformations on the outer surface $R = R_2$ by the formulas:

$$e_{33}(r,t)) = \varepsilon(t)$$
$$e_{22}(r,t) = -\frac{\varepsilon(t)}{2}$$
$$e_{23}(r,t)) = \frac{1}{2}\gamma(t)(1 + (\frac{r_2}{r})^2)\frac{r}{r_2}$$

Local stresses in the sample are associated with an axial force P and moment M by formulas:

$$2\pi \int_{R_1}^{R_2} \sigma_{33} R dR = P(t)$$
$$2\pi \int_{R_1}^{R_2} \sigma_{23} R^2 dR = M(t)$$

The aim of this method is to express the local stresses at the outer surface of the sample through the integral force characteristics, P and M, as a function of the local boundary deformation process $\Pi = (\varepsilon, \gamma)$. In general case, the procedure of the local stress state determination is given in [1, 2]. Below, the basic relations of the method, obtained for the case of a solid circular cylindrical specimen $(R_1 = 0, R_2 = R)$, are provided as an example.

Consider two specimen deformation processes Π and $\Pi + \delta\Pi$ that follow two close kinematic programs $(\varepsilon, \gamma) \equiv (\varepsilon(t), \gamma(t))$ and $(\varepsilon, \gamma(1+h))$, $|h| \ll 1$. Given the proximity of the strain rate in both trials, and, assuming $\Delta P = P(\Pi + \delta\Pi) - P(\Pi)$, $\Delta M = M(\Pi + \delta\Pi) - M(\Pi)$ the following relations can be obtained:

$$\sigma_{33}(\Pi) = \Delta P/(2\pi R^2 h) + 2P/(2\pi R^2)$$
$$\sigma_{23}(\Pi) = \Delta M/(2\pi R^3 h) + 3M/(2\pi R^3)$$
$$\sigma_{11} = \sigma_{22} = 0$$

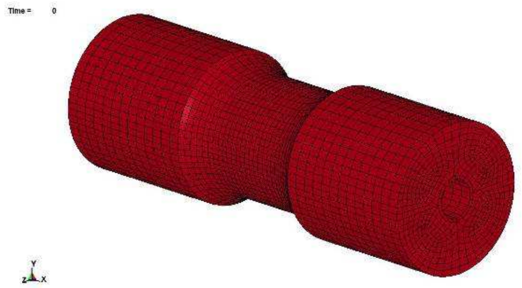
or, given the equality $h = {\Delta\gamma}/{\gamma}$:

$$\sigma_{33}(\Pi) = \frac{1}{2\pi R^2}\left(\gamma\frac{\Delta P}{\Delta\gamma} + 2P\right)$$
$$\sigma_{23}(\Pi) = \frac{1}{2\pi R^3}\left(\gamma\frac{\Delta M}{\Delta\gamma} + 3M\right)$$

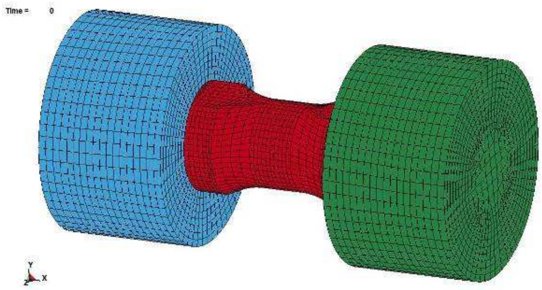
Although, formally, it is required to hold both trials agreed to decrypt each local loading process, we can show that in the case of routine testing, implementing N deformation trajectories of a one-parameter family, no more than (N +1) experiments are required for their decryption. The main obstacle to this approach implementation for dynamic processes is that the kinematic hypotheses used are not obvious. In order to verify their validity, virtual tests using nonlinear LS-DYNA code, modeling biaxial dynamic experiments on the combined compression (tension) and torsion of solid and thick-walled cylindrical specimens by the Split Hopkinson Bar method, were carried out.

Virtual experiments

Virtual experiments were conducted using nonlinear LS-DYNA code. Three types of experiments were simulated with solid and thick-walled cylindrical specimens: torsion of solid specimen, pure torsion of thick-walled specimen and tension-torsion of cylindrical specimen. Geometry for both tests is shown in figures 1a and 1b.



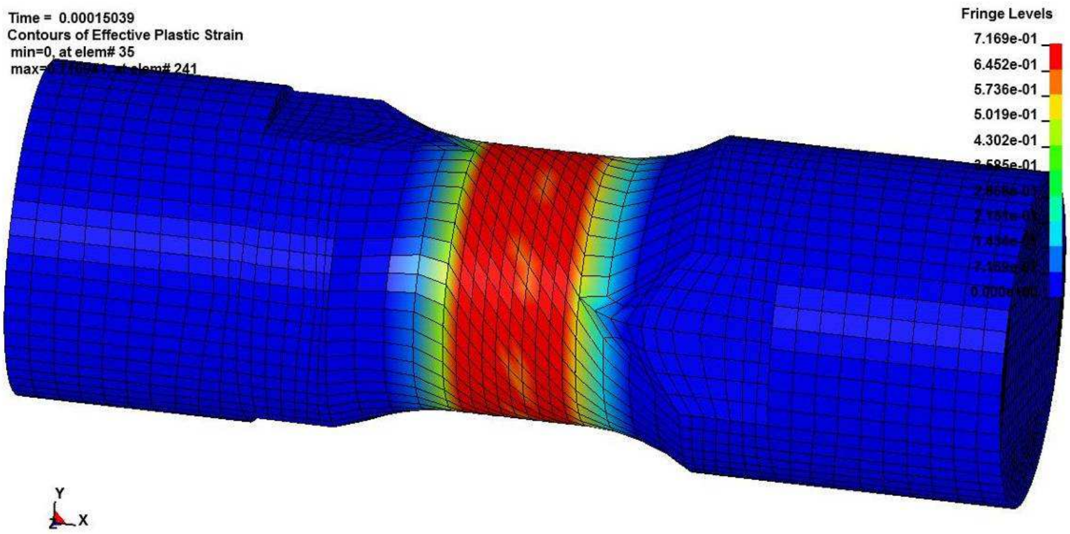
a. Thick-walled cylindrical specimen



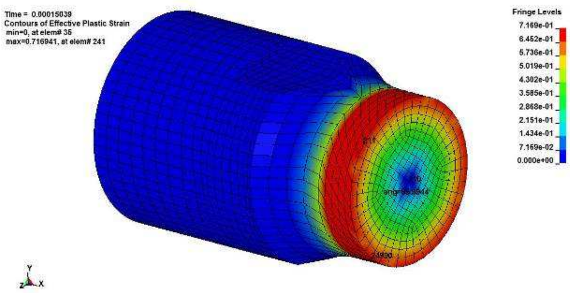
b. Solid cylindrical specimen

Figure 1. Geometry of virtial test benches.

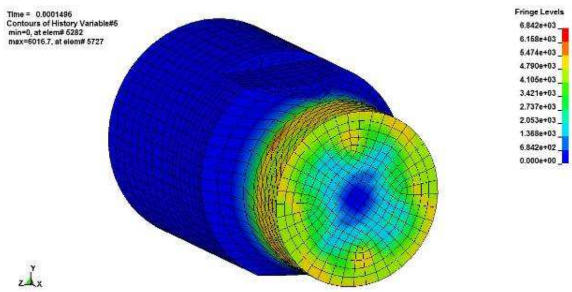
***MAT_224** material model was used in both tests for the structural response simulation. Stress-strain curves were given for different strain rates (800 to 6000 1/s). One end of the specimen was clamped, while the other was subjected to prescribed rotation of 180 degrees in 0.3 ms. Figures below illustrate the validity of the static kinematic hypotheses, showing effective plastic strain, strain rate, triaxiality and Lode angle distributions in solid cylinder test.



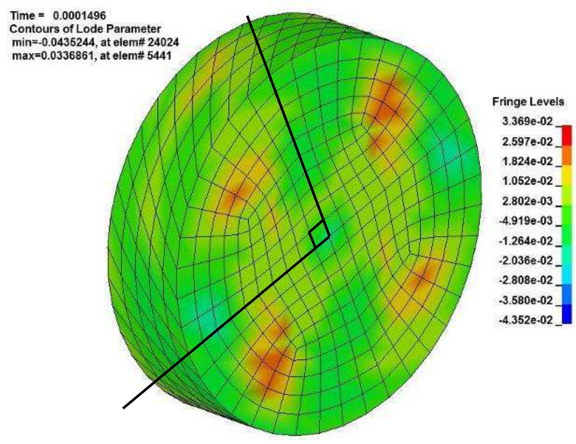
Solid specimen. Plastic strain.



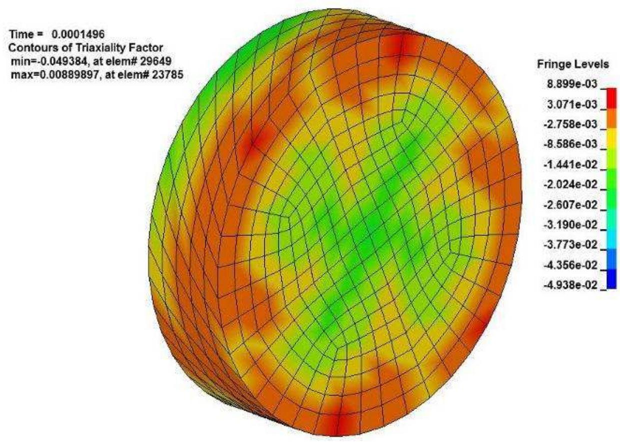
Plastic strain (cut view)



Strain rate



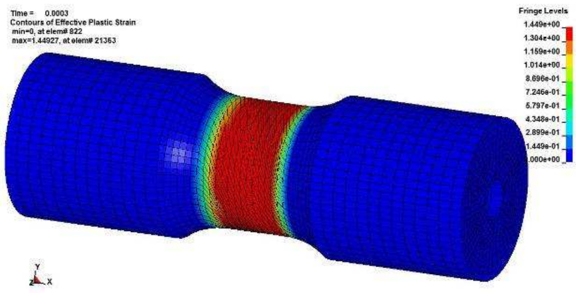
Lode angle



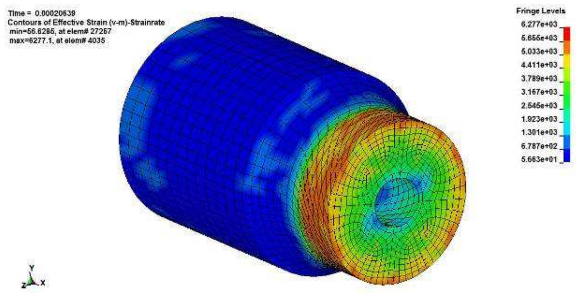
Triaxiality ratio

Figure 2. Solid cylinder test results.

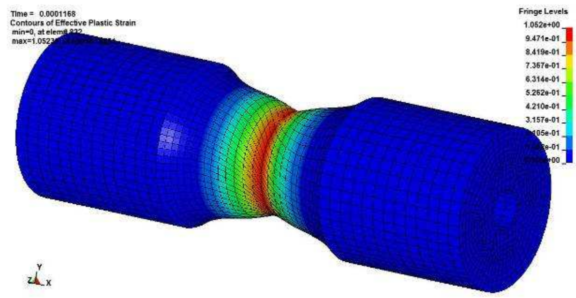
It's seen from the figures that mesh in mid-section is not deformed, having straight radial fibers and rotates in time as a rigid body. That fact confirms kinematic hypotheses, that are known to be valid in statics. It can be also seen that the strain rate in the sample's outer diameter reaches 5000 1/s, while the core remains elastic end statical. The Lode angle and triaxiality are around zero, as for pure shear. Figures below show the same picture for thick-walled cylinder under pure torsion and biaxial experiments.



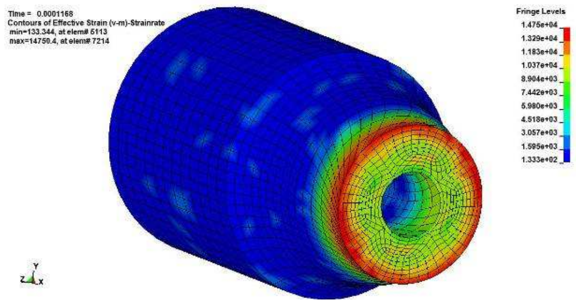
Torsion: plastic strain



Torsion: strain rate



Torsion-tension: plastic strain



Tension-torsion: strain rate

Figure 3. Thick-walled cylinder specimen tests results.

As for solid specimen, it is observed that the mesh in each working part cross-section remains undeformed, while stress state is significantly non-uniformal.

Figure 4 shows the deformation process $(\varepsilon(t), \gamma(t))$ on the outer surface of the specimen for torsion-tension test in time plots.

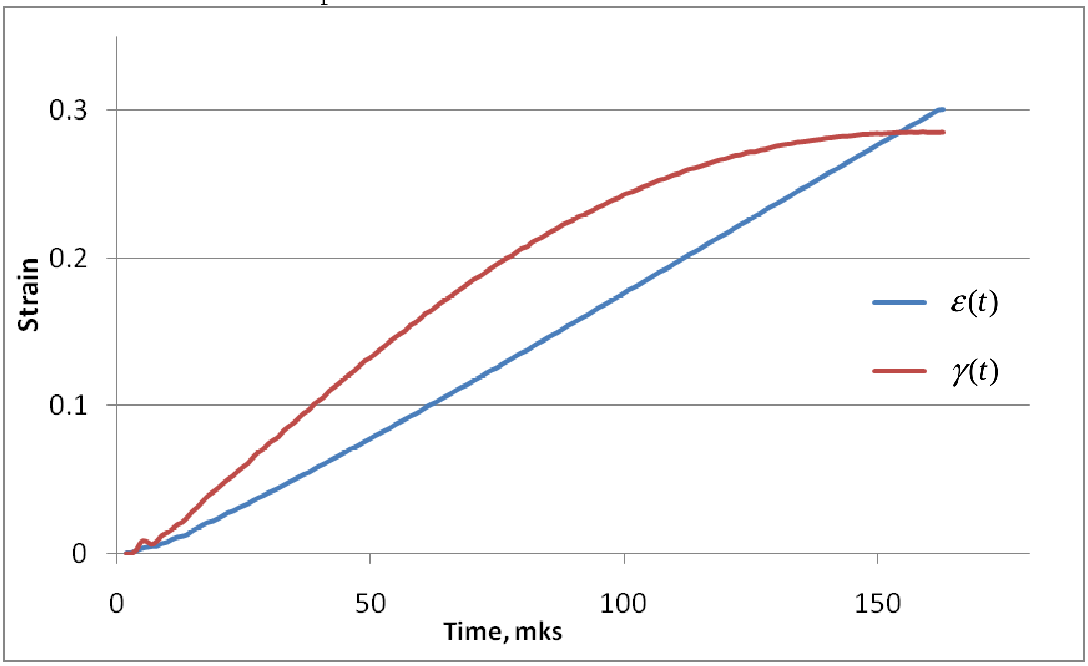


Figure 4. Deformation paths on the outer surface

Thus, the validity of the static kinematic hypotheses is shown throughout the strain rate range, allowed for this method (10^2 - 10^4 s^{-1}).

Conclusion

While studying the failure criteria and constitutive relations of materials under dynamic loading conditions, experimental schemes of combined testing of solid or thick-walled cylindrical specimens in compression (tension) and torsion, useful in statics, can be effective. Using this approach will help to construct the fracture surface dependence on the stress state more accurately. The techniques imaginary tube and degenerate imaginary tube are invited to determine the local stress-strain state in the sample. The validity of taken kinematic hypotheses is justified by the computer simulation on virtual test bench.

References

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