Statistical Analysis of Process Chains :

Novel PRO-CHAIN Components

Daniela Steffes-lai[1,] Tanja Clees[1]

[1]Fraunhofer Institute for Algorithms and Scientific Computing SCAI, Sankt Augustin, Germany

Summary:

The robustness of production processes and the quality of resulting products suffer from variations in important material and process parameters, geometry and external influences, which can have substantial and critical influences. Therefore these variations have to be analyzed and transferred over process steps in order to achieve considerably better forecasting quality.
We developed the PRO-CHAIN strategy for statistical analysis of sensitivity and stability as well as multi-objective robust design-parameter optimization of whole process chains, even for simulation results on highly resolved grids. PRO-CHAIN constructs an ensemble of simulation results; this data base reflects local variations of functionals. Newly developed PRO-CHAIN components deal with transforming and ensemble compression of the data base via a fast principal component analysis with user-controlled accuracy. Essential features are the classification of design parameters into importance and nonlinearity classes in order to reduce the design space and to get an adequate accuracy for an efficient optimization.
In this paper we address the importance of this classification and appropriate kinds of classification measures.
Another main novel PRO-CHAIN component is the fast and accurate interpolation of new designs on the whole grid. This interpolation works also for nonlinear applications like crash if the design of experiments is adequate for a high-quality metamodel. The interpolation is based on a nonlinear metamodel with radial basis functions accelerated by a specialized principal component decomposition.
Summarized, PRO-CHAIN is now able to fully locally analyze a chain consisting of several process steps with regard to sensitivity and robustness and to predict new designs with user-controlled accuracy. In each step, the influence of parameters onto criteria is classified and sensitivity is measured. PRO-CHAIN is able to propagate the essential scatter due to parameter uncertainty locally over the steps, keeping the necessary number of simulation runs small. Additionally, PRO-CHAIN allows for predicting new designs fully locally, allowing for immediate answers to what-if scenarios, without additional time-spending simulation runs. Thus PRO-CHAIN is a very efficient strategy for statistical analysis of process chains, involving parameter uncertainties, in order to get a robustly optimized solution.
Recently, we integrated the efficient interpolation method described into DesParO along with LS-DYNA d3plot readers/writers: on one hand, as a so-called "mixing functionality" for constructing and dumping interpolated results, on the other hand into the novel DesParO Geometry Viewer.
Now, DesParO allows for an interactive exploration of the design space, connected with direct interpolation and visualization of the new design and its functionals, like thickness, effective plastic strains and damages as well as statistical measures, locally on the whole grid.
Results are presented for the forming-to-crash process chain for a ZStE340 metal blank of a B-pillar. In detail, results of importance and nonlinearity classifications in each process step are shown as well as the prediction of new designs by means of DesParO.

Keywords:
Forming, crash, process chain, sensitivity, dimension reduction, statistical analysis, metamodel, interpolation.

1   Motivation

During the fabrication of products, material and process parameters (e.g. damage parameters, friction and forces), geometry and also external influences can vary substantially. These variations can have a substantial, even critical influence on the robustness of production processes and the quality of resulting products. Analyzing governing influences of the variations and possibly minimizing them belongs to the most challenging research and development tasks today. This is especially true for the consideration of whole process chains, for example car parts with a potentially critical influence in crashes, as, for instance, the B-pillar, consisting of several formed and connected blanks.

Thus, there is a high need in the industry for a strategy with coordinated, efficient software modules for statistical analysis with sensitivity and robustness aspects and multi-objective robust design-parameter optimization of whole process chains.

Commonly, the last step of a process is still considered separately. Partly, at least first information from the history is integrated, but without variations. However, considerably better forecasting quality of numerical simulation and optimization can be achieved if not only the history of the process is included in the simulation of the last step as completely as possible, but also variations of decisive parameters are taken into account and transferred over the steps.

The sources of different behavior have to be taken into account as early as possible, that means the pre-history of the process has to be considered within the simulations. Statistical analysis and robust optimization have to start as early as possible in the process. An example of different behavior due to parameter variations is shown in Figure 1.
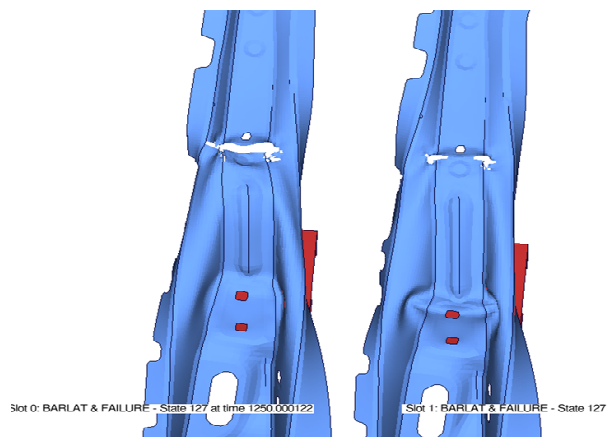

Figure 1: Extreme simulation results (interplay of crack with dent) caused by parameter variations (most relevant one here: d3-10% vs. d3+4%).

In Section 2, we briefly describe the PRO-CHAIN strategy which performs different analysis steps addressing stability, sensitivity, robustness, and optimization aspects. The methods are based on appropriately constructed ensembles of simulation results.

Newly developed methods and strategies for sensitivity analysis and reduction of the design space are discussed in Section 3, especially the characterization of design parameters into nonlinearity and importance classes is presented. In Section 4 we focus on the interpolation and visualization methods with the SCAI software DesParO, which allows for direct preview of results belonging to a new set of design parameters. In Section 5, we demonstrate PRO-CHAIN for a ZStE340 micro-alloyed metal blank of a B-pillar, a decisive part with a potentially critical influence in car crashes. In detail, results of importance and nonlinearity classifications in each process step are shown as well as the prediction of new design by means of DesParO. Section 6 concludes the paper and gives an outlook on future developments.

2   The PRO-CHAIN Strategy

The PRO-CHAIN strategy can be applied in many application areas after a suitable adaptation to specific data formats and robustness criteria, in particular. Exemplary applications in automotive engineering include the forming-to-crash and casting-to-crash process chains, chains of forming steps, forming/casting-to-NVH etc. Applications from the semiconductor industry include the process-to-device and device-to-circuit simulation.

Several components have recently been developed (see [6]) and integrated into PRO-CHAIN that allow for iteratively building, handling and transforming a data base reflecting local variations of functionals (e.g. local thicknesses) even on highly resolved simulation grids. These components can be used for (single processes or) process chains consisting of two and more subsequent processes. Variations to be approximated, characterizing the output of a process step, result from variations of input parameters of the first process step, in particular.

Exemplary, for a basic forming-to-crash process chain, Figure 2, the strategy consists of the following main steps and software tools (in brackets):
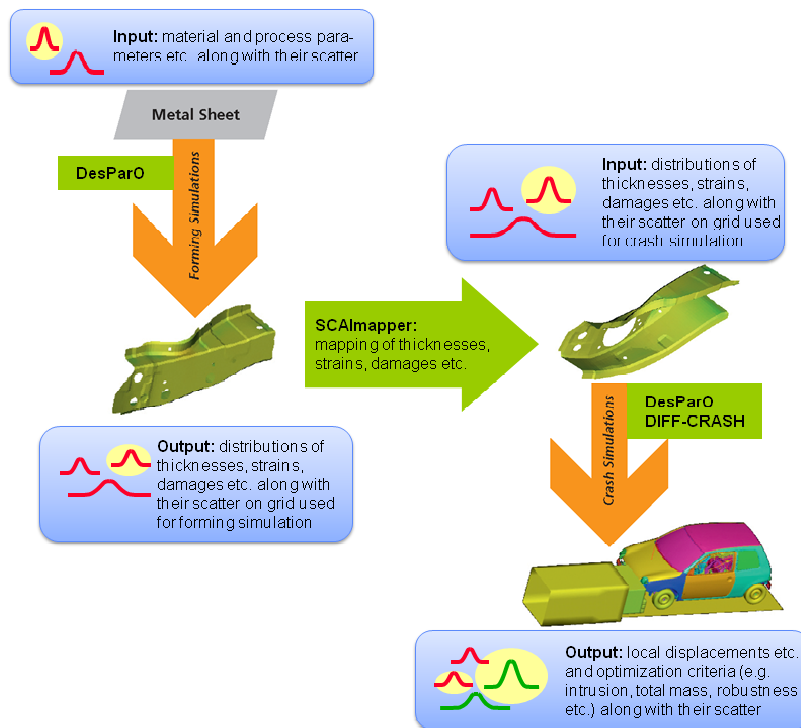

Figure 2 : Process chain forming-to-crash: simulation types (in orange), typical kinds of variations to be dealt with (in blue), software tools (in green) supporting sensitivity and robustness analysis as well as multi-objective robust optimization (DesParO), mapping (SCAIMapper) and a backtracking of instabilities in crash simulations (DIFF-CRASH). For forming and crash simulations, LS-DYNA is employed, for instance. Material models such as IWM's BI-FAILURE can be used.

Analysis of the first process step: Forming scenario
The first step of the process chain considered here is a forming (deep drawing) scenario. The following steps are performed for its analysis:
- Physical experiments (with specimens and components) for obtaining information on parameters for material models and realistic variations (see [1], [8]).
- Setup of the concrete material model to be used (IWM Bi-Failure, see [2], [4], [8]).
- Forming simulations (LS-DYNA). An ensemble of simulations is performed based on a basic design of experiments (DoE), as for instance a center design and lower and upper bounds for variations per parameter.
- Parameter sensitivity analysis and iterative construction of data base (DesParO [12]), see Section 3.
- Comparisons with physical experiments (see [1], [8]) for simulation validation.
- Multi-objective robust design-parameter optimization (DesParO).

In case that simulation results do not live on the same grid, a reference grid has to be specified and the functional values are transferred by means of an interpolation/restriction/approximation method.

Transformation of the database
For transforming the constructed data base (output of the forming scenario) to serve as an input data base for the next step in the process chain, namely the crash scenario, the following two steps are performed:
- Compression of the data base.
- Mapping of the data base (ensemble of special functionals constructed) and setup of the new data base.

The mapping can employ any suitable interpolation or approximation method. Here, we use the SCAImapper [13]. In general, the quality of the mapping has to be measured. Errors resulting from the mapping shall be considerably smaller than the variations resulting from the previous process step.

From the new data base, local thicknesses, effective plastic strains and damages along with their local variations can be reconstructed to be used as inputs for crash simulations. Note that the crash grid is usually coarser than the adaptively refined grid resulting from the forming simulation. In addition, several parts are usually cut from the formed (e.g. deep-drawn) component.

Analysis of the crash scenario
The second step of the process chain considered here is a crash test. The following steps are performed:
- Crash simulations (LS-DYNA).
- Stability analysis (DIFF-CRASH [9], [10], [12]), that means a detailed analysis of instabilities as well as their backtracking.
- Sensitivity analysis and iterative construction of data base (DesParO).
- Comparisons with physical experiments (see [1], [8]).
- Multi-objective robust optimization of the whole process chain (DesParO).

DesParO is a tool for metamodeling, exploration and optimization. Several features have been recently developed, useful for the overall strategy, in particular, fully local sensitivity analysis for sets of design-parameters together with their scatter, more global measures for robustness and correlation, adaptive and hierarchical metamodeling, an improved global-local robust multi-objective optimizer as well as a mixing (interpolation) method for functionals locally on the whole grid, and their visualization, see also Section 4.

3   Sensitivity Analysis and Ensemble Compression

Among the most decisive steps of the overall strategy are the classification approaches [11], [6]. Each parameter is characterized into one or more classes. This step is mostly decisive for the accuracy of the results and the remaining computational effort, because it highly influences the iteratively extended DoE and therefore the number of simulation runs needed in the next process step.

A classification of design parameters into importance classes is necessary to choose the parameter variations which are highly influencing the chosen functional (e.g. damage). In the case that a variation of a certain parameter do not change the functional at least to a certain amount, the respective parameter variation can be omitted. At the other hand, if the variation of a certain parameter highly influences the chosen functional, it is absolutely required to describe the distribution of the parameter as completely as possible and to transfer all information to the next process step, in order to get a high forecasting quality. Additionally we have to investigate the nature of the behavior of parameter variations onto the functional. If the parameter influences the functional in a linear way, 3 simulations will be enough to get a low interpolation error. If the behavior is nonlinear, we need to extend the DoE in order to approximate the functional appropriately.

The classification strategy itself can proceed in several steps. In the first step, parameters which show a linear (or only a slightly nonlinear) and small impact are sorted out. In a second step, parameters showing larger nonlinearities can be characterized in more detail (by means of appropriate additional simulations), prior to an importance classification against the linearly (or slightly nonlinearly) reacting ones. This strategy assures that the number of simulation runs is kept small, while the accuracy, especially at nonlinearly influenced parts of the domain, remains sufficiently high. Additionally the user retains the overall control of both, required accuracy and computational effort, which are, of course, positively correlated with each other.

3.1   Nonlinearity Classes

On the one hand, parameters are classified according to a measure of nonlinearity for an impact of their variations per functional. The measure can be based, for instance, on comparison of the entries of the approximated Jacobian and Hessian matrix (partial derivatives of first and second order of the dependency of a functional on parameter variations), see [6]. Another possible nonlinearity measure can be based, for instance, on the spectral radius of the approximated full Hessian matrix compared with the Jacobian matrix [6].

Parameters can be sorted into two or more classes depending on the concrete measure used. Approaches for decomposing the design space into several domains and applying different measures per domain can be used. This is recommended for getting a higher accuracy in domains which are highly influenced by the application, e.g. the crack and dent areas in crash simulations, see also Section 5.

3.2   Importance Classes

On the other hand, each parameter is classified according to its importance compared to the other parameters at hand. The measure can again be based on values of the Jacobian matrix, for instance, but now by means of a measure which compares congenerous values, see [6]. Again, parameters can be sorted into two or more classes depending on the concrete measure used. Domain decomposing strategies described for nonlinearity classes can be used as well.

3.3   Ensemble Compression

Due to an efficient data processing, huge data bases (i.e. an ensemble consisting of a few hundreds of simulation results on grids with more than one million nodes, several functionals and a few hundreds time steps simultaneously) can be processed quickly on standard multi-core computing equipment. Large random fields can therefore be directly analyzed and global as well as local impacts be detected, [11]. Here, we analyze three output functionals: thicknesses, effective plastic strains and damages.

In addition, the simulation data base can be reduced (ensemble compression) with user-controlled accuracy. Examplary, a principal component analysis (PCA) can be used. Its singular value decomposition (SVD) of the data matrix serves as an ensemble compression, [6]. Together with Parseval's criterion, only the modes necessary for achieving a user-specified tolerance (user-defined norm) are set up. The ensemble compression can be applied in each process step, efficiently reducing computational effort and memory requirements.

4   Interpolation of New Designs

For designing new products and setting up respective production processes, an engineer may want to test and analyze a lot of different designs, resulting from different sets of design parameters, while taking into account variations of each design parameter.

A brand-new component of PRO-CHAIN supports fast and accurate interpolation and visualization even of highly-resolved functionals for new designs along with their local variations. Therefore, PRO-CHAIN helps you to answer a lot of what-if scenarios, without additional time-spending simulation runs. You only have to assure that the set of design parameters is located within the parameter space covered by the previously applied DoE.

The interpolation is based on a nonlinear metamodel with radial basis functions accelerated by a specialized principal component decomposition as used for ensemble compression (Section 3.3). Therefore, the interpolation works well also for nonlinear applications like crash, if the design of experiments is adequate for a high-quality metamodel.

The interpolation procedure has been integrated into DesParO as "mixing functionality" as well as into DesParO's Geometry Viewer. In particular, a reader and writer for the LS-DYNA d3plot format is available. We support ASCII- as well as binary-format files. The mixing functionality of DesParO is very fast due to an efficient data processing: to interpolate a functional consisting of 84,000 nodal values and 100 time steps tooks only 0.35sec on an standard Linux AMD dual core 2.6GHz PC.
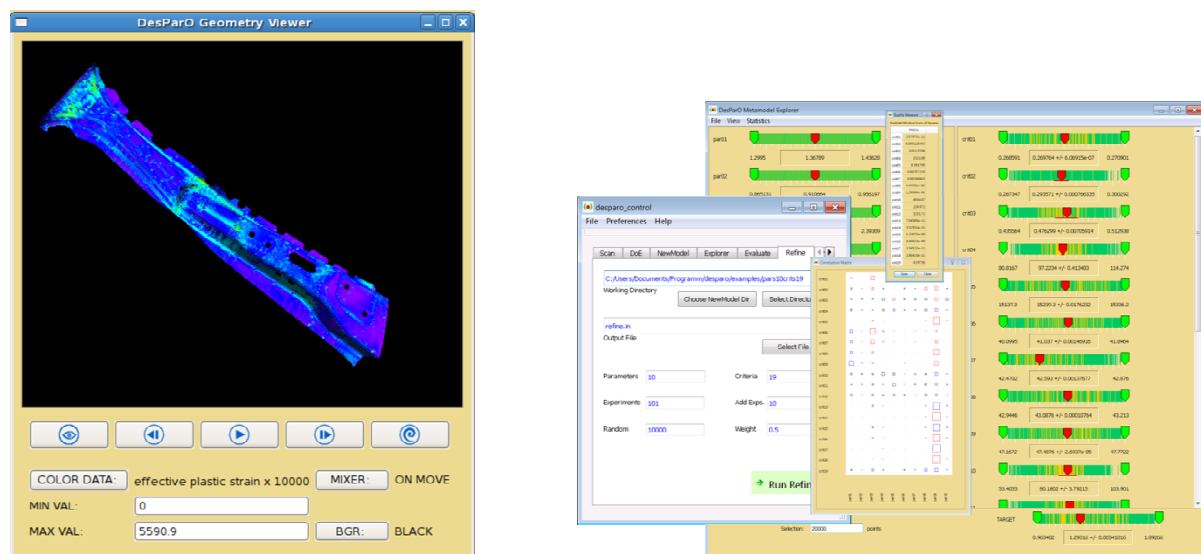

Figure 3: DesParO GUI with novel Control, Explorer and Geometry Viewer components shown. Visualization of effective plastic strain connected with interactive exploration of the design space.

DesParO allows for an interactive exploration of the design space, connected with direct interpolation and visualization of the new design and its functionals, like thickness, effective plastic strains and damages, along with their variations, locally on the whole grid.

5 Results

The PRO-CHAIN strategy has successfully been tested on a ZStE340 micro-alloyed blank of a B-pillar, see [1] - [5] for first discussions and [11] for additional details. Especially detailed results of the novel components of PRO-CHAIN are presented in the following.

The steel sheet with a specified thickness of 1.75mm was characterized with experiments on different specimens [4], [8], and the model parameters were determined.

In the first process step, namely the metal forming (deep drawing), several material and process parameters have been considered. In total, 15 design parameters have been varied for a detailed analysis, see Table 1. The range of variations is also indicated there. The range of values for each parameter reflects variations arising in practice (experimental results).

An experimental validation of the material and damage model was made by means of a component test. To achieve a combination of bending with superimposed tension, the B-pillar was supported at both ends by revolvable bearings. The load application occurred path-controlled (see e.g. [2], [8]).

Table 1: Overview on design parameters along with their range of variations (minima/maxima; distribution not shown) for the metal blank and its forming process.

Parameter Class	Parameters	Range of Variation
damage	d_1, d_2, d_3	±20%
	d_{shear1}, T_{trans}	±20%
hardening	k, n, e_0	±10%
shell thickness	t	±10%
anisotropy coeff.	r_{00}, r_{45}, r_{90}	±10%
friction	μ	±50%
binder force	FORCFN	±10%
drawbead force	DFSCL	±10%

5.1 Numerical results for the forming step

The sensitivity analysis reveals two important material parameters and three important process parameters. The data base can be reduced to a size of three to eight times the size of one simulation result, depending on the desired accuracy. The relevant simulation data for the mapping consist of local distributions of thicknesses, strains and damages. In particular, taking local damages and their variations into account has turned out to be a crucial point in order to achieve simulation results considerably more realistic compared with physical experiments.

The mapping has been carried out by means of the SCAIMapper. Several mapping scenarios have been compared, in order to analyze the effect of taking local distributions of thicknesses, strains and damages of the forming step into account in a step-wise fashion.

The results of the parameter sensitivity analysis confirm the need and the quality of the developed material model. Comparisons with specimen tests show that the novel material model improves the quality of the forming simulations considerably. Comparisons of the component (crash) test results with simulations show that, in particular, including the damage information from the forming step as well as variations of thicknesses, strains, damages caused by parameter variations increases the forecasting quality of numerical simulation considerably (see also [2], [8]). A comparison of the scenarios with and without consideration of thicknesses, strains and / or damages of the forming process, Figure 4, has shown a high influence of the fully mapped data and their variations, especially in critical regions of the B-pillar blank.
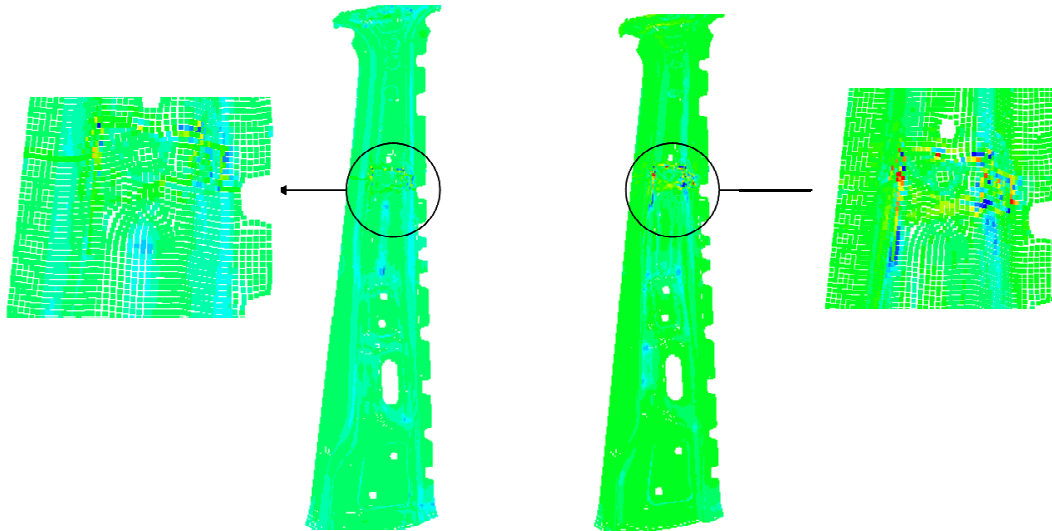
Figure 4: Differences between crash results for the scenarios "without mapping" and "with mapping of local thicknesses, strains and damages", exemplarily for a variation of the d_3 parameter by +20%. Differences in effective plastic strain (left) and differences in damage (right) are shown.

5.2   Numerical results for the crash step

First, a sensitivity analysis of the crash step and an importance ranking as explained in Section 3.2 has been performed. This step adds three parameters (thickness, d2, d3) to the original set of five important parameters (r90, k, process parameters) stemming from the forming step, see Figure 5. Of course, if more parameters are retained in the analysis, a higher accuracy could possibly be reached, but with the drawback of more simulations needed. Here, a global importance measure for the whole B-pillar, based on Jacobian values, has been applied.
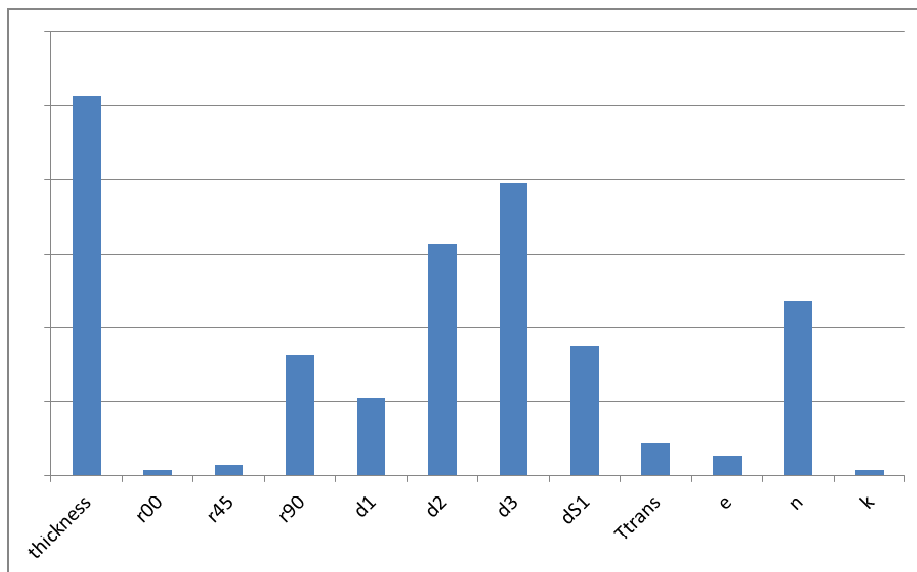

Figure 5: Importance classification of design parameters of the crash step: thickness, d_2 and d_3 are the most influencing ones.

The new methods described in Section 3 allow for a further ranking into nonlinearity classes. In particular, they provide approximations of statistical measures, locally (per node in space and time) on the whole simulation grid, if more simulations are performed and taken into account.

The parameters are classified according to a measure of nonlinearity for an impact of their variations per functional. Figure 6 shows exemplarily the classification of the damage parameter d_3 into nonlinearity classes for an impact of their variations onto the damage functional. The parameter d_3 is one of the most important design parameters, which controls a part of the BI-FAILURE damage model [2]. In detail, Figure 6 shows, that the absolute values of the approximated first derivative (Jacobian matrix) are much smaller than the absolute values of the approximated second derivative (Hessian matrix), especially in the critical regions of the B-pillar. This is also indicated by the applied measure,

which shows highly negative values (blue color) in the critical regions of the B-pillar. Here, negative values indicate a nonlinear influence of the classified parameter onto the functional.
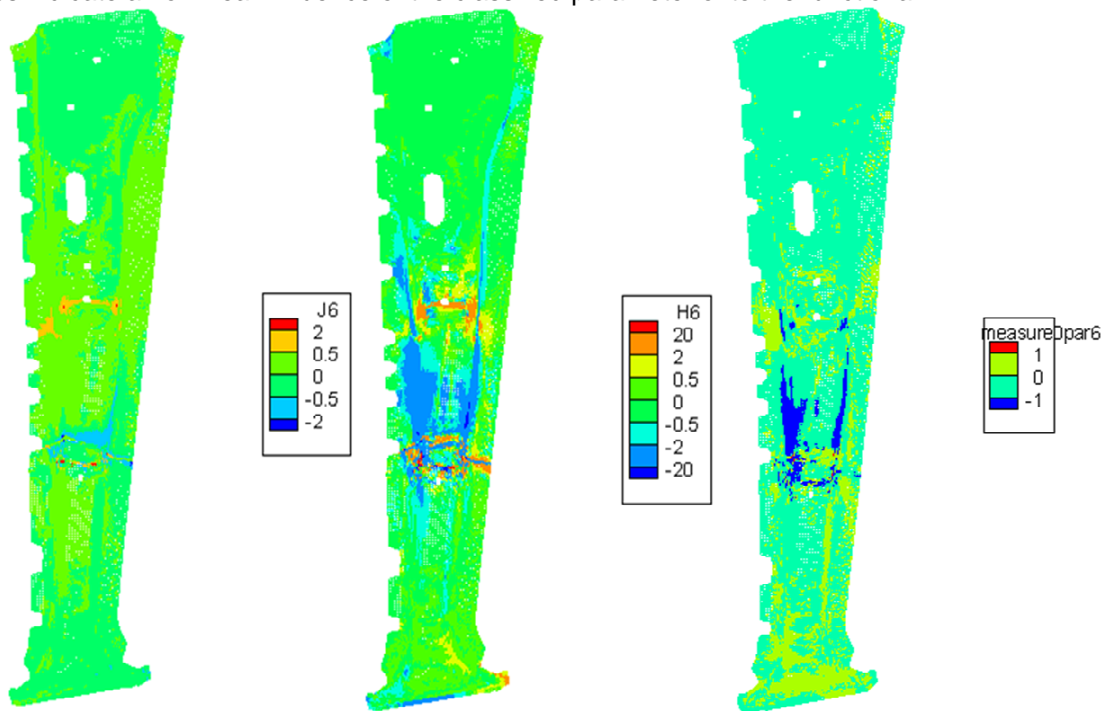

Figure 6: Classification of parameter d_3 into nonlinearity classes. Approximated first derivative w.r.t. d_3 (left), approximated second derivative w.r.t. d_3 (middle), and a measure of nonlinearity (blue color: nonlinear influence) based on the first and second derivatives (right).

Also for the important d_2 parameter a stronger nonlinear behavior is found with the classification measures. Therefore, we will concentrate exemplarily on these parameter d_2 and d_3 in the following, which also have the highest nonlinear influence of the important set of parameters onto the chosen functionals in the crash step.

The force-displacement diagrams resulting from the variation of d_3 and d_2 over the whole forming-to-crash process chain are very different for different simulation runs, which further confirms the nonlinear influences of parameter variations. Figure 7 and Figure 9 illustrate, for instance, the large effect of variations of d_3 and d_2.
Qualitatively similar results are obtained when analyzing other influencing parameters. The entirety of parameter variations yield a force-displacement corridor similar to the one shown in Figure 7. The corridor is not reduced considerably for smaller parameter variations, cf. Figure 9. Even relatively small parameter variations thus influence the crash results substantially here.
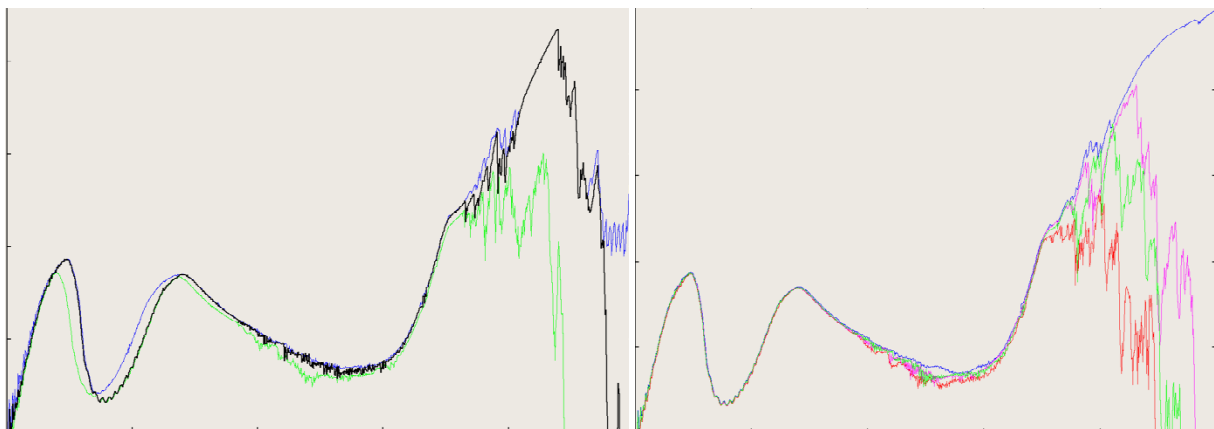

Figure 7: Variation of the d_2 and d_3 parameter, taking the forming history into account, leads to a high variation range of results, as shown in the force-displacement diagrams above. Exemplarily, on the left for the d_2 parameter, minimum per displacement value (green), maximum per displacement value (blue) and initial d_2 are shown, and on the right for the d_3 parameter, d_3 + 20% (pink), initial d_3

(yellow), d_3 - 20% (green), minimum per displacement-value (red), maximum per displacement-value (blue) are shown.

A further sign of the nonlinear influence of the parameters is the different behavior of mean and median, as shown in Figure 8. Therefore, methods taking nonlinearities into account are necessary here, particularly for analyzing critical regions around the crack and dent.
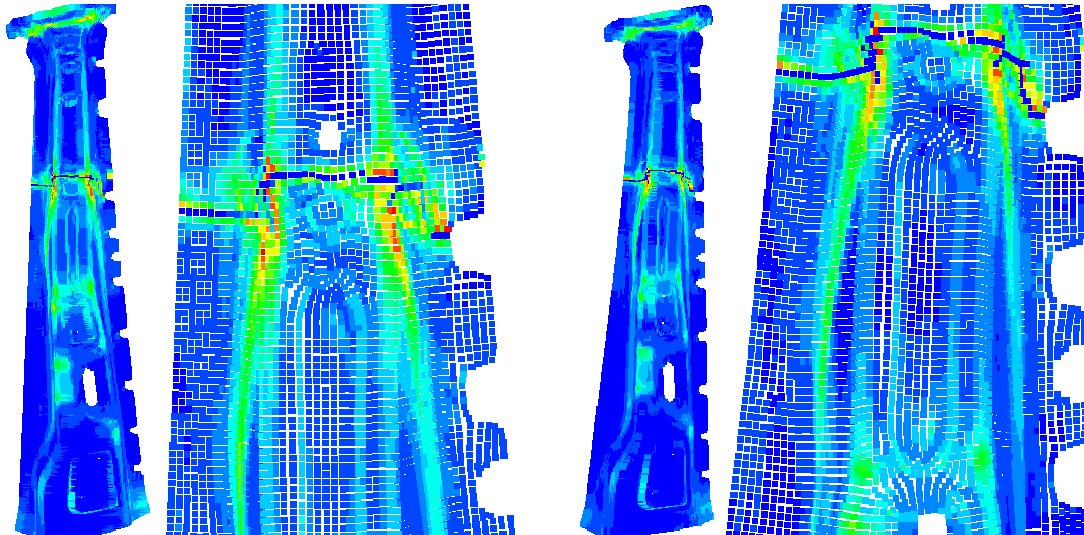

Figure 8: Mean (left) and median (right) of damage values for ensemble of crash simulation results.

Robustness measures derived from statistical functional on the simulation grids should use medians and quantiles instead of means and standard deviations.
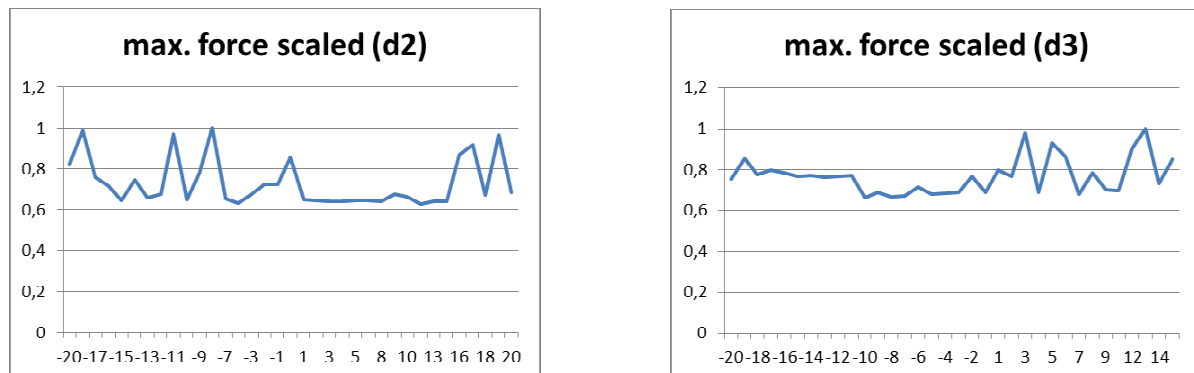

Figure 9: Maximal force, divided by overall maximum, against variation of the d_2 parameter (left), respectively variation of the d_3 parameter (right).

The large corridor arising in the most decisive part of the ensemble of force-discplacement diagrams clearly shows that an optimization process taking global criteria such as maximal force (Figure 9) into account cannot be expected to produce reasonable results here. The development of a novel set of optimization criteria including robustness measures seems necessary.

The interpolation of new designs, described in Section 4, is also tested and validated with simulation runs for several parameter sets. Since even relatively small parameter variations influence the crash results substantially, we show an example of variation of parameter d_2-2% in Figure 10. On the left side the locally interpolated damage values are shown. A simulation run with parameter d_2 varied by -2% has been performed and the obtained results have been compared in order to validate the DesParO mixing functionality. The relative error compared to the simulation result is shown in Figure 10 on the right. The relative error is very small, <1-2%, almost everywhere. Only in single points the error is higher. The interpolation quality can further be improved by taking more simulation runs into account.

The results for the blank can be summarized as follows. Including the pre-damage information from the forming step as well as variations of thicknesses, strains, damages caused by parameter variations increases the forecasting quality of numerical simulation considerably here. Furthermore,

analyzing impacts of these variations, especially with the new classification methods, gives valuable insight into local behavior of the part considered. Additionally the ability of interpolation of new designs turns out to be a fruitful tool in the production process.
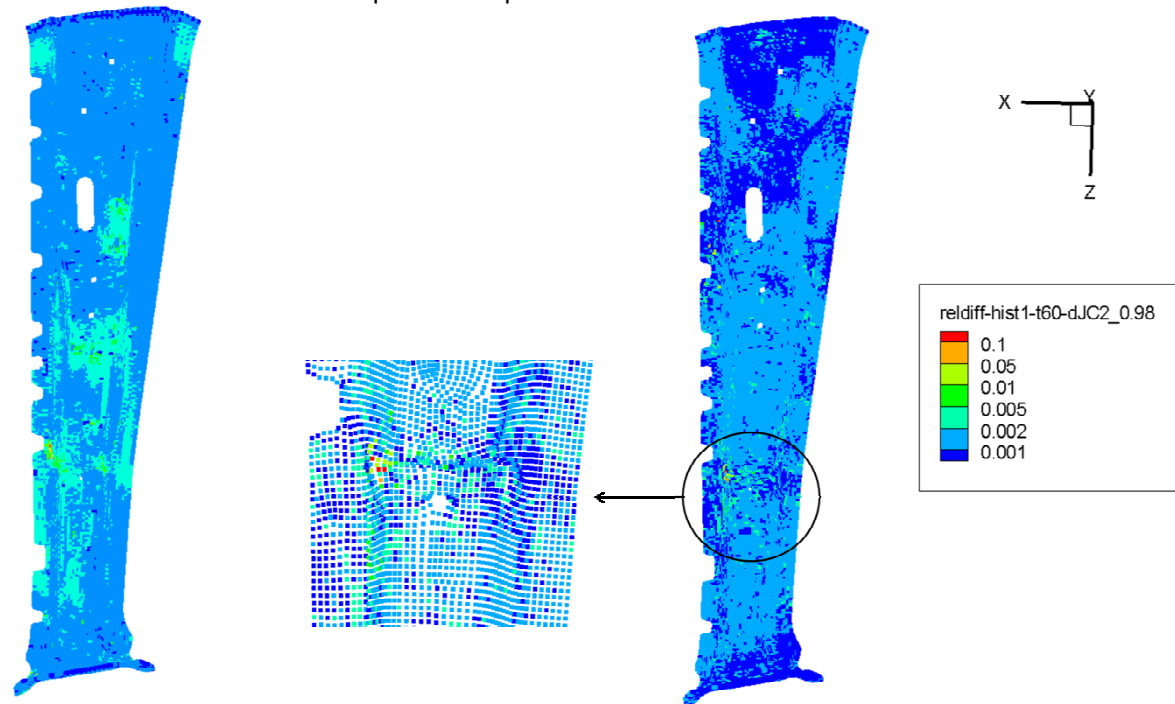

Figure 10: Interpolated functional damage for parameter variation d_2-2% (right) and relative interpolation error compared with crash simulation result.

6   Discussion and Outlook

Newly developed components of the PRO-CHAIN strategy for statistical analysis of sensitivity, stability and robustness aspects have been presented. The obtained results for the ZStE340 micro-alloyed metal blank of a B-pillar demonstrate the efficiency and possibilities of PRO-CHAIN. Important influences of parameter variations were found and their local behavior was characterized. The transfer of local thicknesses, effective plastic strain and damages from the forming to the crash step highly ameliorates the forecasting quality. With the new interpolation method it is easy to check the impact of a certain change in the parameter set. The PRO-CHAIN strategy provides efficient tools for analyzing whole process chains.

In future research we will focus on the one hand on enhancements of the interpolation and visualization methods. For interpolation of new designs, approaches for decomposing the design space into several domains will be investigated in order to be able to choose different resolutions for differently important domains. Interpolation methods can work with such a decomposition if a(n at least) continuous transition is realized by means of decay functions, for instance. Also some work will be done to further accelerate the visualization methods in DesParO. The integration of other formats will be a topic, if desired by industry.
On the other hand, we will further investigate robustness measures and appropriate optimization criteria. A combination of methods described in [7] and [10] for statistical analysis shall be developed.
Additionally, we will apply PRO-CHAIN to more examples from the automotive as well as semiconductor industry.

7   Acknowledgements

8   References

[1] Clees, T., Steffes-lai, D., Helbig, M., Roll, K., Feucht, M.: "Process chain forming to crash: efficient stochastic analysis". Procs. 7th European LS-DYNA Users' Conference, Salzburg, Austria, May 14-15, 2009. Dynamore GmbH, 12 pages.

[2] Helbig, M., Andrieux, F., Steffes-lai, D., Clees, T.: "Development of a material model for analyzing stochastic and deterministic influences on the forming to crash process chain". Procs. ANSYS Conference & 27th CADFEM Users' Meeting, Nov. 18-20, Leipzig, Germany, 2009.

[3] Clees, T., Steffes-lai, D.: "Effiziente stochastische Analyse ganzer Prozessketten mit Anwendungen im Automobilbau". Procs. Symposium "Simulation für robuste Produkte und Prozesse", Eds. Burblies, A., Clees, T., Sauter, J., Bremen, Germany, Feb. 9-11, 2010.

[4] Clees, T., Steffes-lai, D., Helbig, M., Sun, D.-Z.: "Statistical analysis and robust optimization of forming processes and forming-to-crash process chains". 13th ESAFORM Conference on Material Forming, Brescia, Italy, April 7-9, 2010. Int. J. Material Forming 3 (supplement 1), pp. 45-48, 2010. http://dx.doi.org/10.1007/s12289-010-0703-6.

[5] Steffes-lai, D., Clees,T.: "Efficient Stochastic Analysis of Process Chains". Procs. 1st Multiphysics Conf, Bonn, Germany, 2010. Int. J. Multiphysics. Accepted.

[6] Steffes-lai, D., Nikitina, L., Clees, T.: „Vorrichtung und Verfahren zum Bearbeiten einer Prozesssimulationsdatenbasis eines Prozesses". Int. patent application, 2010.

[7] Nikitina, L., Clees, T.: "Apparatus and Method for Analyzing Bulky Data". Int. patent appl., 2010.

[8] Sun D.-Z., Andrieux F., Feucht M.: „Simulation of the process chain from forming to crash taking into account stochastic aspects". Procs. 9th LS-DYNA Users' Conference, Bamberg, Germany, Oct 12-13, 2010. Dynamore GmbH, 10 pages.

[9] Thole, C.-A., Nikitin, I., Nikitina, L., Clees, T.: "Advanced Mode Analysis for Crash Simulation Results". Procs. 9th LS-DYNA Users' Conference, Bamberg, Germany, Oct 12-13, 2010. Dynamore GmbH, 10 pages.

[10] Nikitin, I., Nikitina, L., Clees, T.: "Stochastic analysis and nonlinear metamodeling of crash test simulations and their application in automotive design". In "Computational Engineering: Design, Development and Applications", F. Columbus (Ed.), Nova Science, New York, 2010.

[11] Clees, T., Steffes-lai, D.: "PRO-CHAIN Efficient Statistical Analysis of Process Chains Applied to a Froming-to-Crash Example". Procs. 9th LS-DYNA Users' Conference, Bamberg, Germany, Oct 12-13, 2010. Dynamore GmbH, 11 pages.

[12] www.scai.fraunhofer.de/robust-design.html

[13] www.scai.fraunhofer.de/scaimapper.html

[14] www.carod.fraunhofer.de

[15] www.hiespana.fraunhofer.de