A NEW ADVANCED VISCO-ELASTOPLASTIC EIGHT CHAIN RUBBER MODEL FOR LS-DYNA

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INTRODUCTION

A new advanced eight chain rubber model has recently been implemented in LS-DYNA. The material is tailored for polymeric materials. The basic theory is taken from Arruda's thesis from 1993 but it has been enhanced with advanced features such as the Mullins effect, viscoelasticity, plasticity and viscoplasticity.

The Mullins effect is described by two different models: the first one is strain based and developed by Boyce in 2004 and the second is energy based and developed by Ogden and Roxburgh in 1999.

The viscoelasticity is based on the general Maxwell theory with up to six Maxwell elements (a spring and a dashpot in series).

There are three different viscoplasticity models implemented: a Norton model with two parameters, a G'Sell model with six parameters and a strain hardening model with four parameters. The plastic yield strength is based on the eight parameter Hill model.

The material model has been used to simulate a compression test with a rubber specimen. The material parameters were obtained from inverse FE analys and parameter fitting using LS-OPT and a force-displacement data set. The result shows that this material model can predict rubber behaviour inline with experimental results.

MATHEMATICAL FRAMEWORK

This model is based on the work done by Arruda and Boyce, i.e. in Arruda's thesis from 1993. The eight chain rubber model is based on hyper elasticity and it is formulated by using strain invariants. The theory is based on the split of the deformation tensor $\mathbf{F}$ into an elastic $\mathbf{F}_e$ and a plastic part $\mathbf{F}_p$. From Arruda's thesis, the eight chain model only utilizes the first invariant $I_1 = tr(\mathbf{C}_e)$ where $\mathbf{C}_e = \mathbf{F}_e{}^T\mathbf{F}_e$ is the right Cauchy Green deformation tensor. The strain softening is taken from work done by Boyce 2004, where the strain energy used is defined as

$$\Psi = v_s \mu \left[\sqrt{N}\Lambda_c \beta + N \ln\left(\frac{\beta}{\sinh\beta}\right)\right] + K\Psi_2 \quad = \Psi_1 + \Psi_2 ,$$

where the amplified chain stretch is given by $\Lambda_c = \sqrt{X\left(\overline{\lambda}^2 - 1\right) + 1}$ and

$$\beta = L^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right),$$

where $\overline{\lambda}^2 = I_1/3$, μ is the shear modulus of the soft domain, N is the number of rigid links between crosslinks of the soft domain region, v_s is a saturation parameter and

$X = 1 + A(1-v_s) + B(1-v_s)^2$ is a general polynom describing the interaction between the soft and the hard phases (Boyce 2004 and Tobin and Mullins 1957).

The compressible behavior is described by the strain energy part:

$$\Psi_2 = \frac{1}{v_{con}}\left(v_{con}\ln J + \frac{1}{J^{v_{con}}} - 1\right)$$

where v_{con} is a compressibility parameter and J is the determinant of the deformation gradient.

The Cauchy stress is computed as:

$$\boldsymbol{\sigma} = \frac{2}{J}\mathbf{F}_e\frac{\partial\Psi}{\partial\mathbf{C}_e}\mathbf{F}_e^{\,T} = \frac{1}{J}\mathbf{F}_e(\mathbf{S_1}+\mathbf{S_2})\mathbf{F}_e^{\,T} = \frac{v_s X\mu}{3J}\frac{\sqrt{N}}{\Lambda_c}L^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)\left(\mathbf{B}_e - \frac{1}{3}I_1\mathbf{I}\right)$$
$$+\frac{2K}{Jv_{con}}\left(1-\frac{1}{J^{v_{con}}}\right)$$

Where $\mathbf{S_1}$ and $\mathbf{S_2}$ are the second Piola-Kirshhoff stresses based on Ψ_1 and Ψ_2, respectively.

Mullins effect

Two different models for the Mullins effect are implemented. Firstly the model described by Boyce 2004 is a strain driven softening model. The evolution of the softening is described by the following equation:

$$\dot{v}_s = Z(v_{ss} - v_s)\frac{\sqrt{N}-1}{\left(\sqrt{N}-\Lambda_c^{\max}\right)^2}\dot{\Lambda}_c^{\max},$$

where Z is a parameter that characterizes the evolution in v_s with increasing $\dot{\Lambda}_c^{\max}$. The parameter v_{ss} is the saturation value of v_s. Note that $\dot{\Lambda}_c^{\max}$ is the maximum of Λ_c from the past:

$$\dot{\Lambda}_c^{\max} = \begin{cases} 0 & \Lambda_c < \Lambda_c^{\max} \\ \dot{\Lambda}_c & \Lambda_c > \Lambda_c^{\max} \end{cases}.$$

It means that the structure evolves with the deformation. The dissipation inequality requires that the evolution of the structure is irreversible $\dot{v}_s \geq 0$ (see Boyce 2004).

The second model available for the Mullins effect is based on work done by Ogden and Roxburgh 1999. When activated, the strain energy is a standard eight-chain (Arruda-Boyce) model. That is, the following parameters are automatically set: $Z = 0, v_s = 1$ and $X = 1$. The stress is a multiplicative split between the virgin response and the softening parameter:

$$\overline{\boldsymbol{\sigma}} = \eta\boldsymbol{\sigma}$$

where

$$\eta = 1 - \frac{1}{m_1}\operatorname{erf}\left(\frac{\Psi_1^{\max} - \Psi_1}{m_3 + m_2\Psi_1^{\max}}\right)$$

and m_1, m_2 and m_3 are material parameters. $\Psi^{\max}$ is the maximum strain energy that has been achieved in the loading path.

Viscoelasticity

Two viscoelastic models are available. Firstly we have a model where the viscoelasticity is based on a generalized Maxwell model described in Holzapfel (2000). The evolution equation for the in-equilibrium stresses has the form

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\beta_\alpha \frac{d}{dt}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \beta_\alpha \dot{\mathbf{S}}_\mathbf{1}$$

where α is the number of viscoelastic terms (max 6). The evolution is integrated and solved for each time step and the total second Piola-Kirshhoff stress is given as

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \sum_{\boldsymbol{\alpha}=\mathbf{1}}^{\mathbf{6}} \mathbf{Q}_{\boldsymbol{\alpha}}$$

Secondly we have added viscoelastic model whose evolution equation is based on Simo and Hughes (2000) and renders

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\frac{\gamma_\alpha}{\tau_\alpha}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \frac{\gamma_\alpha}{\tau_\alpha}\mathbf{S_1}$$

where α is the number of prony terms (max 6), $\gamma_\alpha \geq 0, \tau_\alpha > 0$. The equation is integrated and solved by a recursion scheme and the total second Piola-Kirchhoff stress is given by

$$\mathbf{S} = \gamma_\infty \mathbf{S_1} + \mathbf{S_2} + \sum_{\boldsymbol{\alpha}=\mathbf{1}}^{\mathbf{6}} \gamma_\alpha \mathbf{Q}_{\boldsymbol{\alpha}}$$

where $0 \leq \gamma_\infty < 1$ and

$$\gamma_\infty = 1 - \sum_\alpha \gamma_\alpha$$

When the second Piola-Kirchhoff stress has been calculated the total Cauchy stress is obtain by a standard push forward operation:

$$\boldsymbol{\sigma} = \frac{1}{J}\mathbf{F}_e \mathbf{S} \mathbf{F}_e{}^T .$$

Plasticity and Viscoplasticity

The plastic relation is based on the general Hills' yield criterion

$$f = \sigma_{eff} - \sigma_{yld} \leq 0$$

where

$$\sigma_{eff}^2 = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{12}^2 + 2M\sigma_{23}^2 + 2N\sigma_{13}^2$$

and the hardening is either based on a load curve ID (-YLD0) or an extended Voce hardening

$$\sigma_{yld} = \sigma_{yld0} + \sum_{i=1}^{4} Q_i \left(1 - e^{B_i \bar{\varepsilon}}\right).$$

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For the viscoplastic phenomenon, we simply add one evolution equation for the effective plastic strain rate. Three different evolution laws are available.

- A simple Norton model with two parameters and where the effective plastic strain rate is given by:

  $$\dot{\varepsilon}_{vp} = \left(\frac{f}{K_1}\right)^{S_1}.$$

  The yield criterion $f \leq 0$ gives the final equation to solve

  $$f - \dot{\varepsilon}_{vp}^{1/S_1} K_1 = 0$$

  where $\dot{\varepsilon}_{vp}$ is the effective plastic strain rate and K_1 and S_1 are viscoplastic material parameters.

- A G'Sell model with six parameters and where the effective plastic strain rate is given by:

  $$\dot{\varepsilon}_{vp} = \left(\frac{f}{K_1\left(1-e^{-S_1\left(\bar{\varepsilon}_{eff}+K_2\right)}\right)e^{S_2\bar{\varepsilon}_{eff}^{K_3}}}\right)^{S_3}.$$

  The yield criterion gives the final equation to solve:

  $$f - K_1 e^{S_2\varepsilon_{eff}^{K_3}}\left(1-e^{-S_1\left(\bar{\varepsilon}_{eff}+K_2\right)}\right)\dot{\varepsilon}^{1/S_3} = 0$$

  where K_1, S_1, K_2, S_2, K_3 and S_3 are viscoplastic material parameters.

- A strain hardening model with four parameters and where the effective plastic strain rate is given by:

  $$\dot{\varepsilon}_{vp} = \left(\frac{f}{K_1}\right)^{S_1}\left(\bar{\varepsilon}_{eff}+K_2\right)^{S_2}$$

  The yield criterion gives the final equation to solve:

  $$f - K_1\left(\frac{\dot{\varepsilon}_{vp}}{\left(\bar{\varepsilon}_{eff}+K_2\right)^{S_2}}\right)^{\frac{1}{S_1}} = 0.$$

Kinematic hardening

The kinematic hardening is based on the effective plastic strain whereas the plastic deformation is obtained from the plastic rate of deformation

$$\mathbf{D}_p = \mathrm{sym}\,\mathbf{L}_p = \dot{\varepsilon}_{vp}\frac{\boldsymbol{\sigma}_{dev}}{\left|\boldsymbol{\sigma}_{dev}\right|}$$

where the rate of the plastic deformation gradient is given from the definition of the plastic velocity gradient:

$$\mathbf{L}_p = \mathbf{F}_p^{-1}\dot{\mathbf{F}}_p \Rightarrow \dot{\mathbf{F}}_p = \mathbf{F}_p\mathbf{L}_p.$$

Without loss of generality we assume that the antisymmetric part of the velocity gradient is included in the elastic deformation. The update formula for the plastic deformation therefore renders

$$\mathbf{F}_p^{n+1} = \mathbf{F}_p^n + \Delta t\mathbf{F}_p^n\mathbf{D}_p^n$$

The back stress is now calculated similar to the Cauchy stress above but without the softening factors:

$$\boldsymbol{\beta} = \frac{\mu_p}{3J} \frac{\sqrt{N}}{\Lambda_c} L^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)\left(\mathbf{I} - \frac{1}{3} I_p \mathbf{C}_p^{-1}\right)$$

where $\mathbf{C}_p = \mathbf{F}_p^T \mathbf{F}_p$, $I_p = \mathrm{tr}(\mathbf{C}_p)$ and μ_p is a hardening material parameter. The total Piola-Kirschhoff stress is now given by $\mathbf{S}^* = \mathbf{S} - \boldsymbol{\beta}$ and the total stress is given by a standard push forward operation with the elastic deformation gradient $\mathbf{F}_e$.

EXPERIMENTS

The investigated rubber material was Trelleborg Industrial Rubber, material # 9038703, with hardness 72±5 Shore. A series of tests was conducted where cylindrical test specimens were subjected to axial and radial compression tests, shear tests, and combined axial compression and shear tests. Only the axial compression tests are considered in the present study.

The compression specimen according to ISO 7743:1989 consists of a cylinder with a diameter of 30 mm and a height of 12.5 mm. The cylinder is compress between two parallel highly polished flat metal plates. In order to accomplish an approximate homogeneous state of deformation the flat surfaces were lubricated with Teflon.

Each specimen was first loaded and unloaded to 50% compression (λ/λ_0=-0.5) in ten cycles. A loading speed of 400 mm/min was applied in this pre-loading, and it was found that the effect from the loading speed on the subsequent test results could be neglected.

The subsequent axial compression tests were conducted at three loading speeds, i.e. 40 mm/min, 80 mm/min, and 400 mm/min. Each axial compression test was repeated five times. A typical load deformation test result is shown in Figure 1.

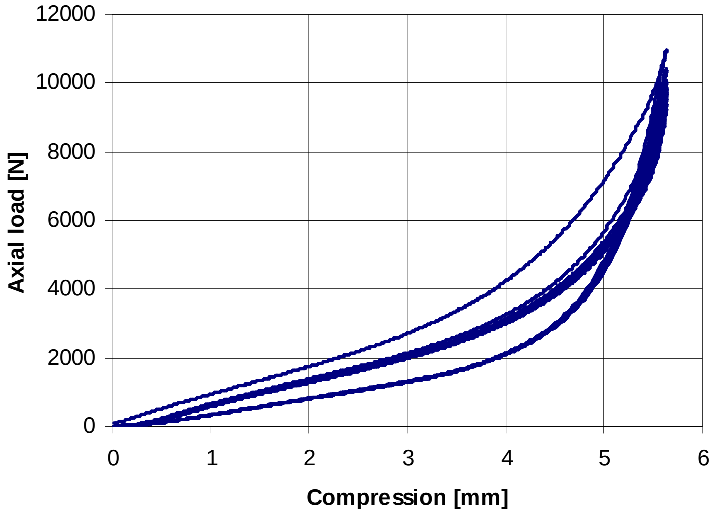

Figure 1: Cyclic load of rubber specimen. Loading speed 400 mm / min.

PARAMETER IDENTIFICATION AND VALIDATION

This model was calibrated against a real compression test using LS-DYNA and the optimization software LS-OPT v.4.1. As described in the previous section the specimen where compressed 50%, which corresponds to a displacement of 5.7 mm and a compression force of 1.0 kN.

The simulation and optimization where done on a 2D axi-symmetric model with two rigid plates and one rubber specimen. The rubber specimen with geometry 13.8x12.7 mm (height x length) in its initial state, were compressed into 18.5x7.0 mm which can be seen in Figure 2. The two plates (not shown) were treated with Teflon to minimize the friction between the plate and the rubber.

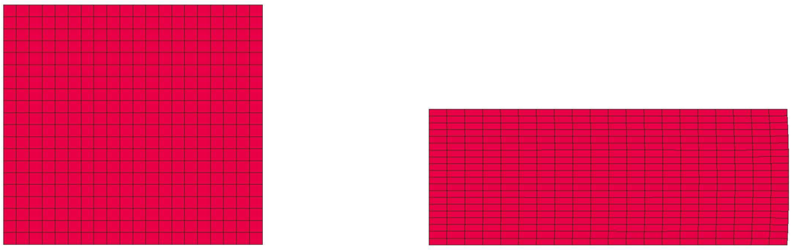

Figure 2: The initial configuration to the left and the final compressed state to the left.

The behaviour of the rubber specimen where assumed to exclude viscoelastic, viscoplastic and plastic phenomena. Seven parameters were chosen for the optimization: the bulk modulus, the shear modulus, the number of cross-links, the static friction between the plates and the rubber, and the three parameters that are included in the Ogden-Roxburg Mullins model. The optimal configuration where achieved with LS-OPT in 14 iterations while trying to minimize the experimental force required to compress the specimen with the force calculated from LS-DYNA. The optimal parameters can be seen in Table 1.

Variable	Value
Bulk modulus (K)	1.81 GPa
Shear modulus (μ)	1.15 MPa
Number of crosslinks (N)	32
Static friction coefficient	0.01
Mullins parameter 1 (M1)	1.35
Mullins parameter 2 (M2)	0.10
Mullins parameter 3 (M3)	0.72

Table 1: Optimum set of parameters for the rubber compression test.

The force-displacement plot in Figure 3 shows a good agreement with the experimental values. Note that the experimental values are scaled down to fit the axi-symmetric model (thereof the much lower forces than in Figure 1). The last half of the loading path fits very well with the experiment and the unloading path is almost spot on. However, the current model configuration is unable to capture the effect early in the load path, which discrepancy may be corrected by activate the strain driven Boyce Mullins effect and viscoelasticty.

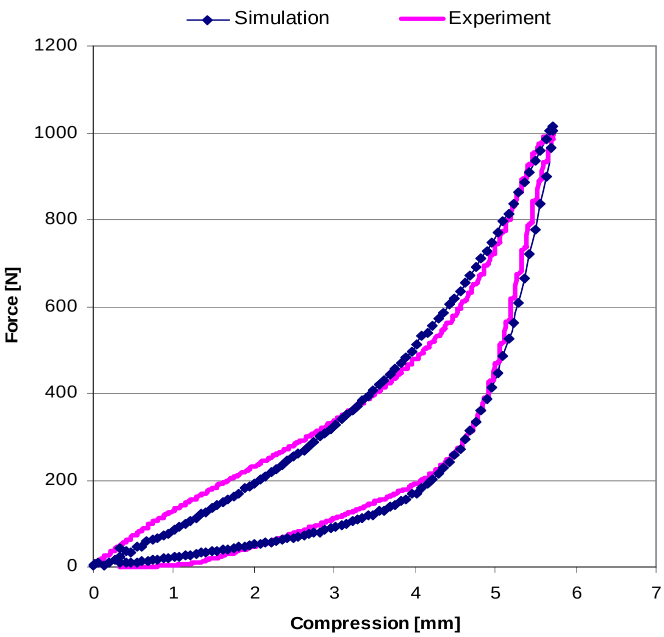

Figure 3: Force-displacement curves from the experimental test and the LS-DYNA simulation. The experimental values are scaled to fit the axi-symmetric simulation model.

References

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