Complexity based design robustness analysis

Application to mechatronic component (vehicle hatchback)

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Definitions

- Let Model be **f**=**f**(**A**,**<u>X</u>**)
  - Let **A** = Model parameters
  - Let **X**= Model Variables

- Parametric analysis -> variations of **A**

- Sensitivity analysis -> variations of **X**

- Optimization -> best choice of **X** = $\mathbf{X}_o$

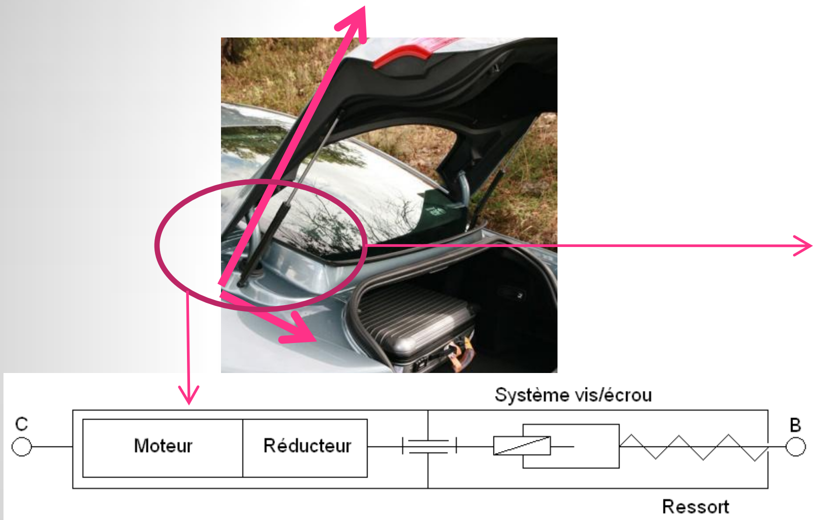
- Model dispersion -> Stochastic analysis -> **~A**

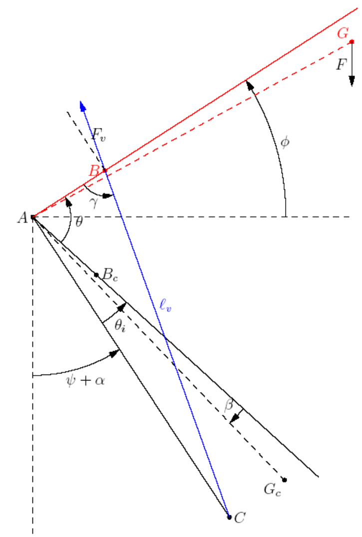
- Robust optimization -> best choice $\mathbf{X}_{ro}$ –>F(f, σ(**X**,**A**))

- Complexity based robust optimization ?

Objectives

- Establish robustness indicators for the optimal design of electric motor





MOVEO/O2M project: Courtesy of Valéo and SupMéca

- Expected results:
  - Establish optimal design variables subject to given model parameters
  - Identify optimal layout solution taking into account the robustness of the solution
  - Identify impact of uncertainty in model parameters on optimal design

Criteria and constraints

- Minimize the maximum motor power needed: Min(MaxPm en W)

- Subject to:
  - cstr0 ≥ 0 (minimum lever length) $l_v^{init} \geq l_{min}$
  - cstr1 ≥ 0 (maximum lever path) $l_v^{max} - l_v^{init} \leq l_v^{init} - l_{mr}$
  - cstr2 ≥ 0 (Minimum spring length) $l_v^{init} - l_{mr} \geq dn + Sa$
  - cstr3 ≥ 0 (Openning time) $p_v r \geq \frac{2\pi(l_v^{max} - l_v^{init})}{t_m \omega_m}$
  - cstr4 ≥ 0(Spring internal diameter) $D - d \geq 0.02$

« Optimal Design » proccess

- Optimization of electric hatchback motor (NLPQL)
  - Define and determine optimal design variables
  - Define criteria related to vehicle cost and performance
  - Define vehicle and environment parameters (assumed fix)

- Monte Carlo Analysis (LH)
  - Variables set to optimum
  - Variation of vehicle parameters
  - Variation of environmental parameters

- Robustness analysis
  - Identification of system fragility (complexity based robustness)

Model variables

- Variables of motor model

-	ω_m :	angular velocity (tr/min)
-	$G_{ressort}$:	spring shear module (Pa)
-	Lv_{min} :	minimum arm length of jack (m)
-	Lmr :	length of (geared) motor (m)
-	η_1 :	motor efficiency (S-U)
-	η_2 :	geared motor efficiency (S-U)
-	η_3 :	efficiency of screw system (S-U)
-	n :	Number of spring coils (S-U)
-	d :	spring cable diameter (m)
-	D :	mean spring diameter (m)
-	p_r :	spring increment (m)
-	p_v :	screw increment (m)
-	Inv_r:	Transmission ratio (S-U)

- variable min/max bounds:

-	ω_m =	3209.09 tr/min	$2700 < \omega_m < 3300$
-	$G_{ressort}$ =	$8.27575*10^{10}$ Pa	$8.0*10^{10} < G_{ressort} < 8.3*10^{10}$
-	Lv_{min} =	0.20141 m	$0.18 < Lv_{min} < 0.22$
-	Lmr =	0.15561 m	$0.135 < Lmr < 0.165$
-	η_1 =	0.7525	$0.7 < \eta_1 < 0.9$
-	η_2 =	0.7626	$0.7 < \eta_2 < 0.9$
-	η_3 =	0.8475	$0.7 < \eta_3 < 0.9$
-	n =	48	$40 < n < 55$
-	d =	0.003828 m	$0.003 < d < 0.004$
-	D =	0.023535 m	$0.022 < D < 0.03$
-	p_r =	0.010798 m	$0.009 < p_r < 0.011$
-	p_v =	0.011818 m	$0.006 < p_v < 0.012$
-	Inv_r =	27	9 < Inv_r < 36

- **Note :** Model Variables **X** => (assume known if not use model reduction techniques based on machine learning techniques developed by CADLM)

Model parameters

- <u>Vehicle parameters</u>
  - X_a ; Z_a : point A (axis X et Z) (m)
  - X_b ; Z_b : point B (axis X et Z) (m)
  - X_c ; Z_c : point C (axis X et Z) (m)
  - X_g ; Z_g : point G (axe X et Z) (m)
  - t_m: maximum openning time (s)
  - θ: hatchback openning angle (°)
  - Masse: hatchback mass (kg)
  - δ_{LB} : distance variation B1/B2 (m)
  - δ_{LC} : distance variation C1/C2 (m)

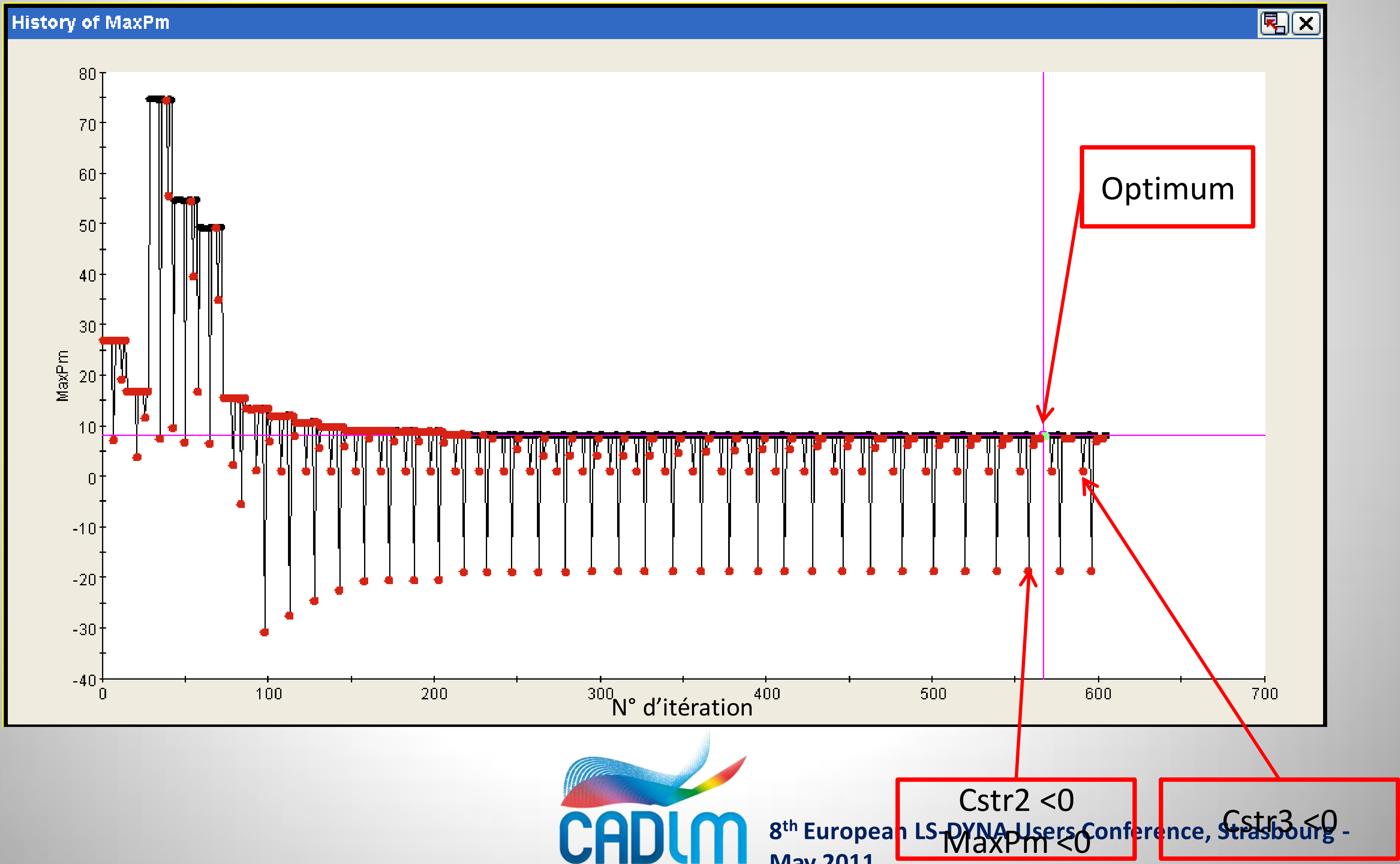
- Vehicle:
  - X_a = 0.34135 m ; Z_a = 1.68229 m
  - X_b = 0.44169 m ; Z_b = 1.59108 m
  - X_c = 0.65209 m ; Z_c = 1.20849 m
  - X_g = 0.73909 m ; Z_g = 1.26679 m
  - t_m = 7.08 s
  - θ = 83.72°
  - Masse = 31.9 kg
  - δ_{LB} = 0.008601 m
  - δ_{LC} = 0.02 m

- <u>Environement parameters</u>
  - α: angle of hatchback to horizontal (°)
  - g: gravity (m/s^2)

- Environnement :
  - α = 0.0°
  - g = 9.81 m/s^2

Optimization process

- Minimize (MaxPm) :



Optimal layout

- System optimal variables:
  - ω_m = 3290.71 tr/min
  - $G_{ressort}$ = $8.3*10^{10}$ Pa
  - Lv_{min} = 0.22 m
  - Lmr = 0,165 m
  - η_1 = 0,8411
  - η_2 = 0,84
  - η_3 = 0,84
  - n = 55
  - d = 0.004 m
  - D = 0,024 m
  - p_r = 0.011 m
  - p_v = 0.00635463 m
  - Inv_r = 14

- Optimal output (all constraints >0):
  - **MaxPm = 8,05 (W)**
  - cstr0 = 0,228432 (m)
  - cstr1 = 0,108162 (m)
  - cstr2 = 0,0526323 (m)
  - cstr3 = 2,52733E-6 (m)
  - cstr4 = 4,85723E-17 (m)

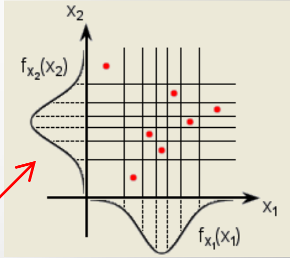
- Iterations: **605**

Analyse Monte Carlo

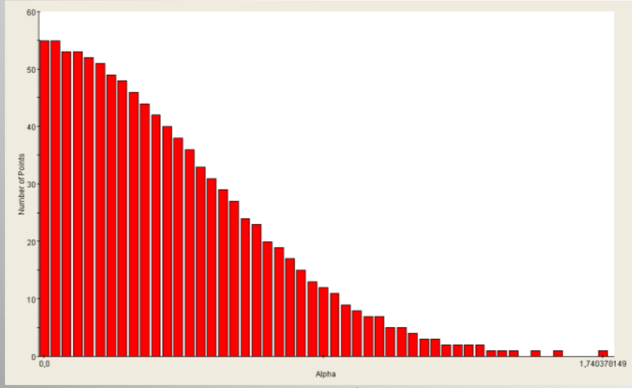
- « Destabilize » system around optimal

- Assume normal distribution of model parameters around given bounds:

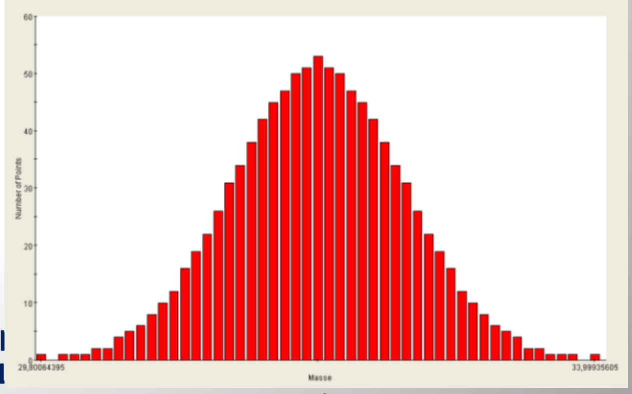
  - $\alpha_{nominal} = 0.0°$ — $0.0° < \alpha < 1{,}74°$ — (≈ slope of 3%)
  - $Masse_{nominal} = 31{,}9$ kg — $29{,}8$ kg $< Masse < 34{,}0$ kg — => +/- 2kg
  - $\theta_{nominal} = 83{,}72°$ — $83{,}706° < \theta < 83{,}734°$ — => +/- 0,02°
  - $Xg_{nominal} = 0{,}73909$m — $0{,}734$m $< Xg < 0{,}744$m — => +/- 5mm
  - $Zg_{nominal} = 1{,}26679$m — $1{,}264$m $< Zg < 1{,}269$m — => +/- 3mm
  - $tm_{nominal} = 7{,}08$s — $7{,}01$s $< tm < 7{,}15$ — => +/- 0,07s



- 1001 points



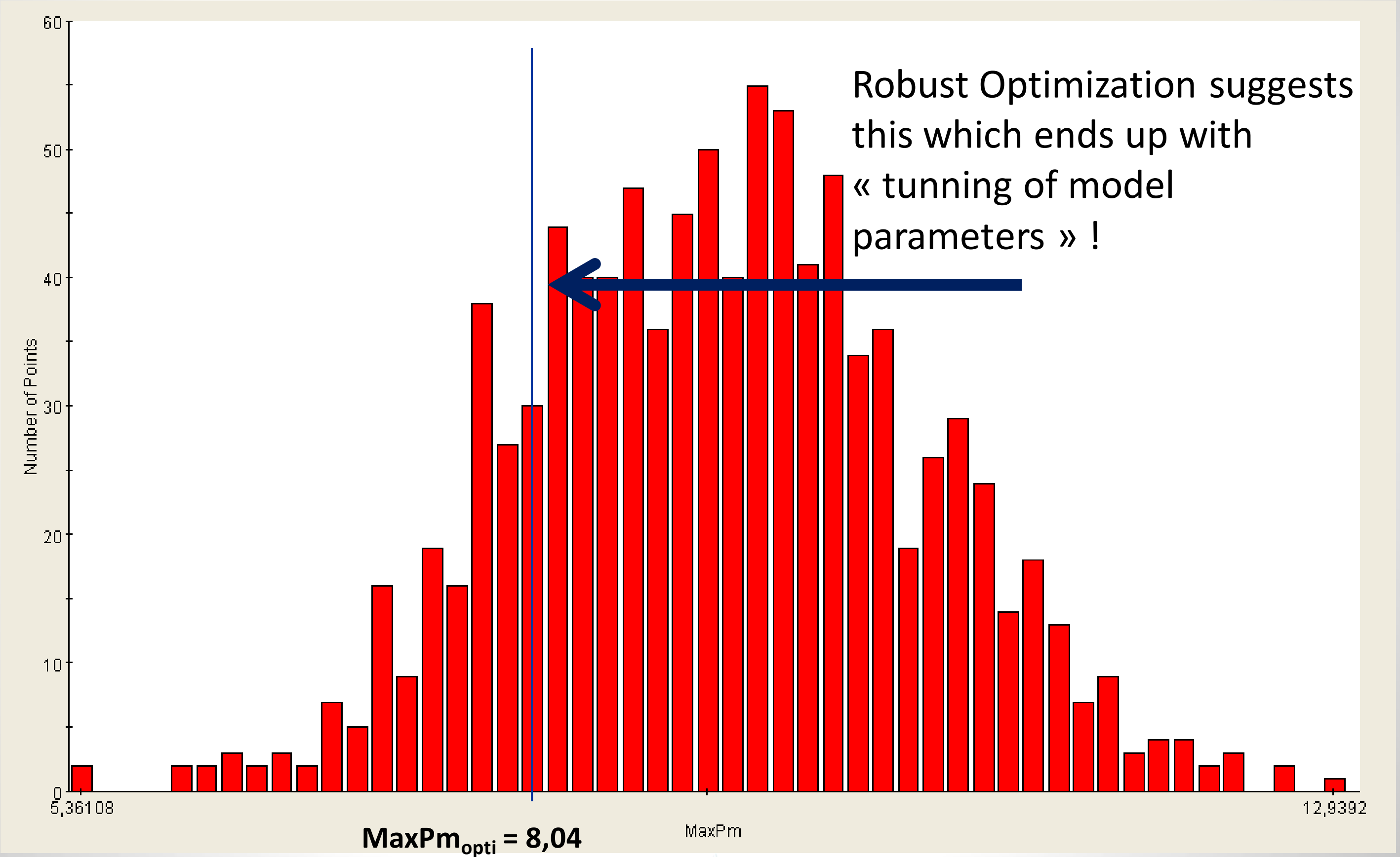
Variation de α



Variation de Masse

Monte Carlo based dispersion

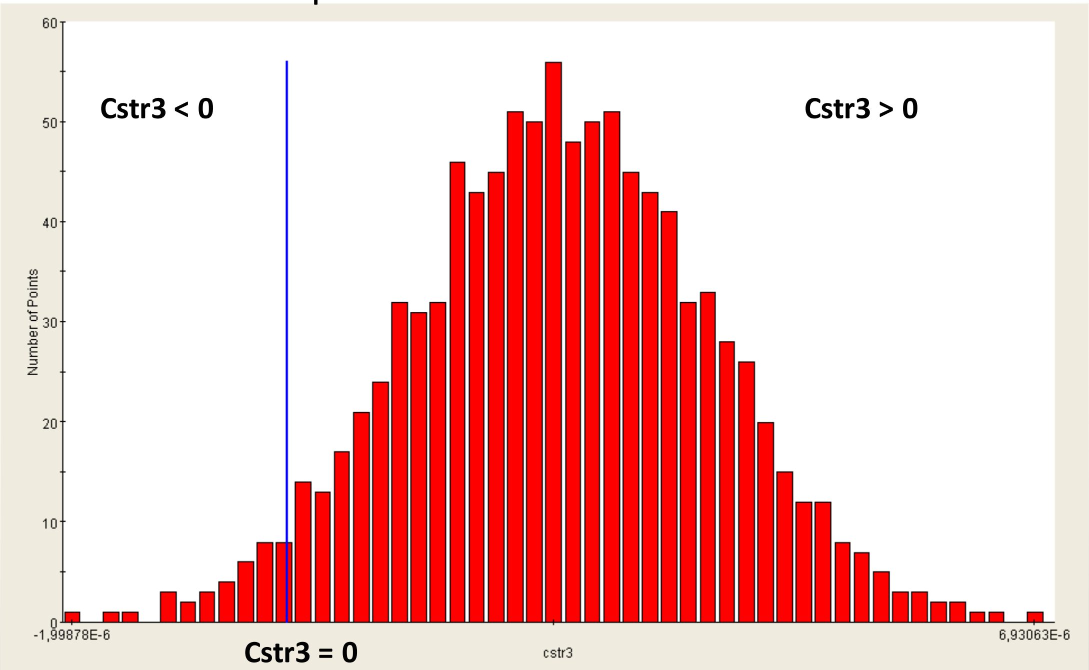
- Motor power variation (970 points / 1001 satisfy constraints)



Analyse Monte Carlo

- Constraint satisfaction

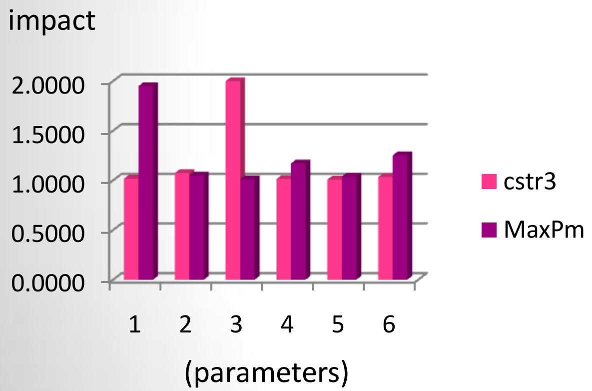
  - cstr0 : **100%**
  - cstr1 : **100%**
  - cstr2 : **100%**
  - cstr4 : **100%**
  - cstr3 : **97%** => critical for « optimum » robustness

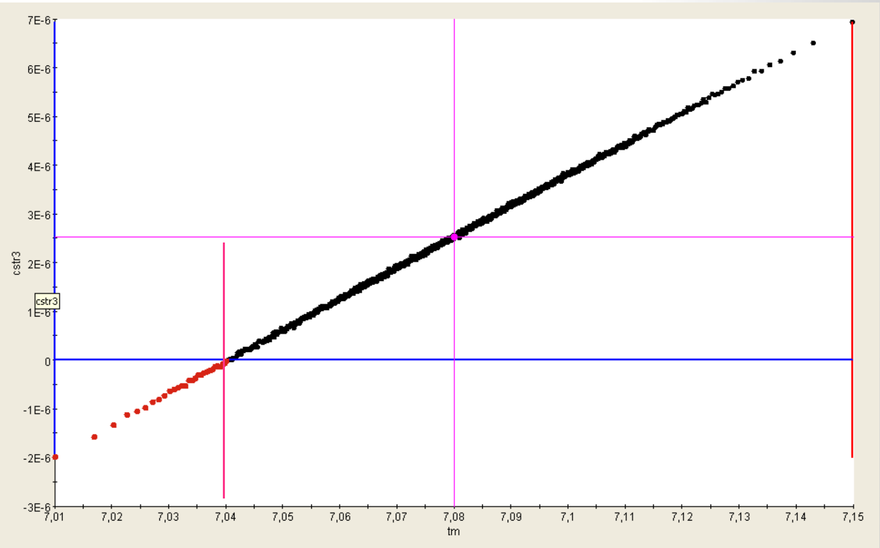


Conclusion (results)

We observe:

- ➢ Constraint cstr3 is violated due to tm





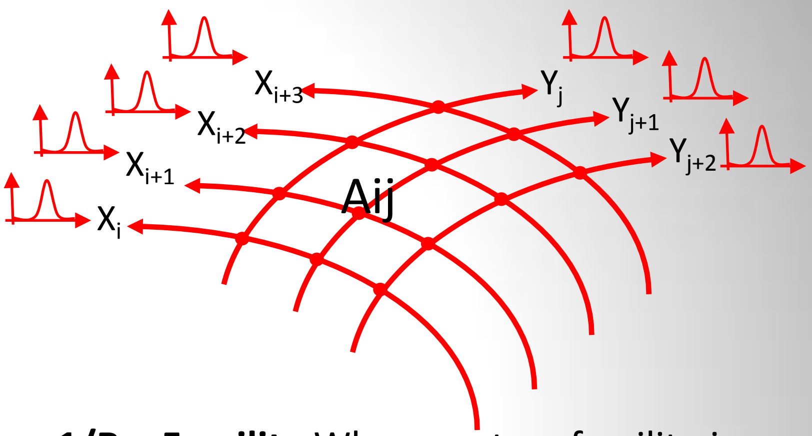
Tm_{limite} = 7.04

- ➢ Dispersions of MaxPm are due to Masse

From stats to robustness?

- Is this true?
- Can we say anything more?

Robustness indicators (CADLM)



- Definition:
  - **Robustness is a measure of "surprise" or "non-"Gaussian" behaviour of the system**

Let **R** define the robustness of the system defined as => **1/R = Fragility** Where system fragility is a product (combination) of system **Complexity** x **Uncertainty**

In it's most simple manifestation **complexity is an indicator representing system topology (and cross-correlation).**

Each system parameter **A** includes its uncertainty as well as it's impact on the whole system. Identification of these relations allows us to establish **a complexity map of the system which leads to system « weakest elements »**

In general **entropy based complexity indicators are very good candidates**. However, any other (simplified indicator could already provide a first insight into the problem).

A simple definition of fragility

• Uncertainty of each parameter/response (around optimum) $\boldsymbol{\lambda_j}$ may be defined as:

$$\lambda_j = 2 - \frac{Y_{j-opti}}{Y_{j-opti} + \left\| Y_{j-opti} - Y_{j-moy} \right\| + Y_{j-max} - Y_{j-min}}$$

•Complexity contribution of **each parameter** may be defined as a function of

- $\boldsymbol{\alpha_1}$ -> parameter correlation with each response
- $\boldsymbol{\alpha_2}$ -> parameter own uncertainty **(=λi)**
- $\boldsymbol{\alpha_3}$ -> P:arameter cross-correlation with other parameters

Where each one of the above may be obtained via $\mathbf{C_{xy}}$ **= absolute value of Bravais-Pearson coefficient :**

$$r_p = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

σ_{xy} = covariance between x et y
σ_x *et* σ_y = standard deviation of x and y

Robustness analysis

- System correlation matrix

Matrice de corrélation	Masse	Theta	tm	Xg	Zg	Alpha	cstr0	cstr1	cstr2	cstr3	cstr4	MaxPm
Masse	2	1.0402	1.0183	1.0077	1.0601	1.0067	1.0000	1.0399	1.0000	1.0173	1.0000	1.9497
Theta	1.0402	2	1.0555	1.0527	1.0197	1.0048	1.0000	1.9996	1.0000	1.0742	1.0000	1.0489
tm	1.0183	1.0555	2	1.0151	1.0079	1.0335	1.0000	1.0561	1.0000	1.9998	1.0000	1.0113
Xg	1.0077	1.0527	1.0151	2	1.0100	1.0132	1.0000	1.0537	1.0000	1.0142	1.0000	1.1714
Zg	1.0601	1.0197	1.0079	1.0100	2	1.0624	1.0000	1.0196	1.0000	1.0082	1.0000	1.0388
Alpha	1.0067	1.0048	1.0335	1.0132	1.0624	2	1.0000	1.0045	1.0000	1.0336	1.0000	1.2530
cstr0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	2	1.0000	1.0000	1.0000	1.0000	1.0000
cstr1	1.0399	1.9996	1.0561	1.0537	1.0196	1.0045	1.0000	2	1.0000	1.0747	1.0000	1.0488
cstr2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	2	1.0000	1.0000	1.0000
cstr3	1.0173	1.0742	1.9998	1.0142	1.0082	1.0336	1.0000	1.0747	1.0000	2	1.0000	1.0102
cstr4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	2	1.0000
MaxPm	1.9497	1.0489	1.0113	1.1714	1.0388	1.2530	1.0000	1.0488	1.0000	1.0102	1.0000	2

Xi: Masse, Theta, tm, Xg, Zg, Alpha — Yj: cstr0, cstr1, cstr2, cstr3, cstr4, MaxPm

each value is a measure of coefficient of Bravais-Pearson.

Robustness analysis

•

•Parameter/response correlations

α_1 =Cij	Masse	Theta	tm	Xg	Zg	Alpha
cstr0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
cstr1	1.0399	1.9996	1.0561	1.0537	1.0196	1.0045
cstr2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
cstr3	1.0173	1.0742	1.9998	1.0142	1.0082	1.0336
cstr4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
MaxPm	1.9497	1.0489	1.0113	1.1714	1.0388	1.2530

•Parameter uncertainty measure

α_2	Masse	Theta	tm	Xg	Zg	Alpha
λi	1.1163	1.0003	1.0194	1.0130	1.0033	1.6814

•Vecteur de corrélation de chaque variable avec les autres

	Masse	Theta	tm	Xg	Zg	Alpha
α_3	1.0266	1.0346	1.0261	1.0197	1.0320	1.0241

Robustness analysis

•Parameter complexity contribution

Yj

Ai

Matrice de complexité	cstr0	cstr1	cstr2	cstr3	cstr4	MaxPm
Masse	1.0476	1.0609	1.0476	1.0534	1.0476	1.3642
Theta	1.0116	1.3448	1.0116	1.0364	1.0116	1.0279
tm	1.0151	1.0338	1.0151	1.3484	1.0151	1.0189
Xg	1.0109	1.0288	1.0109	1.0157	1.0109	1.0680
Zg	1.0118	1.0183	1.0118	1.0145	1.0118	1.0247
Alpha	1.2352	1.2367	1.2352	1.2464	1.2352	1.3195

- $C_{ij} = 1$ => low uncertainty or low correlation with response or other parameters
- $C_{ij} = 2$ => large uncertainty / strong correlation with response or other parameters

Robustness analysis

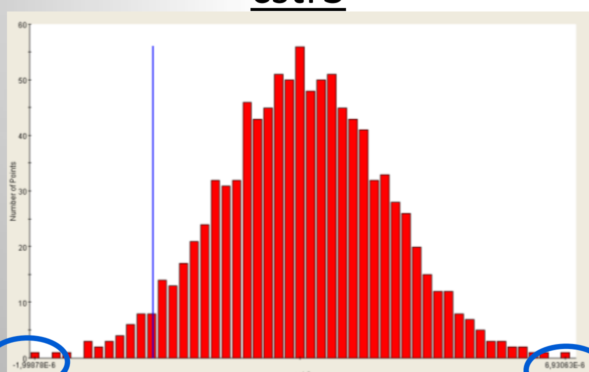
•System response uncertainty

Réponses du système	Incertitude
cstr0	1.000
cstr1	1.001
cstr2	1.000
cstr3	1.779
cstr4	1.000
MaxPm	1.521

Yj

Large uncertainty
(λj les + proches de 2)

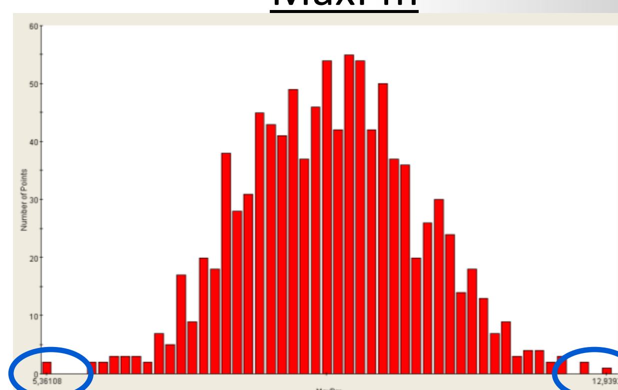
cstr3



-2.00e-6

6.93e-6

MaxPm



5.36

12.94

Robustness analysis

- Impact factor (influence) of parameters on system fragility

Xi

Vehicle/environment parameters	0 < Fragility< 1
Masse	0.359
Theta	0.298
tm	0.338
Xg	0.249
Zg	0.236
Alpha	0.528

Alpha is the most influent parameter (since it's influence is not symetric therefore large uncertainty)

Masse follows since MaxPm strongly dependent on it paramètre et MaxPm a une incertitude importante

tm est le troisième plus influent car cstr3 est quasi corrélé qu'avec tm et cstr3 a une incertitude importante

Conclusion

AVANTAGES

- Much more practical and less iterations than robust optimization (no specific tools required)
- Complexity based robust analysis is superior since it provide information on uncertainty and topology of the system
- Only MC like analysis, no approximation methods (DOE or RSM, FORM, SORM, etc.)
- Allows for definition of easily measurable real-time (static or dynamic) system complexity indicators
- Simple implementation uses Bayesian statistics

INCONVENIENTS

- needs distinguish X and A
- needs good knowledge of A (usage, vehicle, environment)
- Full implementation needs introduction of entropy based indicators (information content, etc.)