On Adaptive Finite Element Analysis in Structural Dynamics of Shell-Like Structures – A Specific View on Practical Engineering Applications and Engineering Modelling

Karl Schweizerhof[*#] and Stephan Kizio*

*Institut für Mechanik, Universität Karlsruhe (TH)
DYNAmore GmbH, Stuttgart
76128 Karlsruhe, Germany

Summary:

Adaptive Finite element analysis in structural analysis has reached a fairly mature status; however, in practice only fairly little is visible in engineering applications. This contribution gives an overview over some major aspects with a specific focus on structural dynamics of shell-like structures.

Adaptive analysis in structural mechanics is mainly focusing on static problems where a large number of linear and nonlinear tasks in 3D continuum problems as well as in shell problems have been tackled. The developed procedures show really considerable improvements for the executed numerical problems. In statics on one side the competition among the methods is between low and high order approximation – the h- and p- or hp-enhancement and among error estimation between global and local estimation. For a good overview over the subject for a large number of problems it is referred to [11], for some mathematical background see [2]. For some simple benchmark problems the differences between the various approaches as presented in [5].

More recent developments are concerned with time dependent problems thus we are focusing here on structural dynamics [10]. In statics - linear or nonlinear – the spatial error distribution is the dominating quantity whereas in dynamics the consideration of the spatial error distribution over the complete considered time range and as well the consideration of the error due to time integration is needed. Here the consideration of dual problems – known in statics from the Betti-Maxwell principle and extended here with the according reciprocity idea, the Graffi-theorem [1] - allows checking the error in specific quantities at certain points in time, the so-called goal-oriented error computation or local error computation [3] [4]. On the basis of such error estimations, in principle, the adaptive modification of the finite element mesh as well as the time step is possible.

However, while in a semi-discretization approach the time step could be fairly easily adjusted – which is frequently done in the so-called explicit FE programs using the central difference scheme - the modification of the finite element mesh introduces major problems. First the data have to be properly mapped between meshes avoiding non-physical artifacts and second the dual error estimation scheme has to take into account different time steps and meshes. Both actions introduce further errors into the analysis which can hardly be judged. In addition the effort for the numerical analysis concerning the computation as well as the required storage becomes overly large [6] leading to the conclusion that adaptive analysis of real world problems based on dual error estimation cannot be handled - at least with the current computer environment.

Thus the focus of this contribution is on a discussion first on the importance of different parts of the error estimation and on the adaptive procedure and second how the major ingredients of the adaptive duality based analysis for practical engineering problems - restricting to shell problems - can still be used, regaining efficiency [7]. For some classes of shell type problems some simplifications can be suggested while still improving the quality of the analysis considerably by adaptive procedures [8].

In structural dynamics also eigenmodes and eigenvalues are important, thus improvements concerning these are also briefly discussed [9] [8]. Obviously the dominating quantity for achieving good results applying finite element methods in structural mechanics is a consistently refined mesh;

not unexpected for high frequency excitations and interest of the engineers in these almost uniformly refined meshes with high mesh densities are required.

It is shown, how the developed schemes can be applied to homogeneous problems and the limits concerning real world engineering models which include a large number of violations concerning standard continuum mechanics are presented.

Also the procedures implemented in LS-DYNA [12], [13] for adaptive analysis are discussed with the background set above. Further some model adaptivity for large structural computations where some parts are – at least in some early states of the analysis – hardly deforming. This effect can be used to introduce rigid bodies in the analysis; the question then arises, how this can be handled.

References

[1] J.D. Achenbach. *Reciprocity in Elastodynamics*. Cambridge University Press, 2003.

[2] W. Bangerth and R. Rannacher. *Adaptive Finite Element Methods for Differential Equations*. Birkhäuser Verlag, 2003.

[3] R. Becker and R. Rannacher. A feed-back approach to error control in finite element methods: Basic analysis and examples. *E.-W. J. Num. Math.*, 4:237-264, 1996.

[4] R. Becker and R. Rannacher. An optimal control approach to a posteriori error estimation in finite element methods. *Acta Numerica*, 10:1-102, 2001.

[5] M. Bischoff; K. U. Bletzinger; A. Düster; S. Kizio; J. Neumann; E. Rank; E. Ramm and K. Schweizerhof. Comparison of different FE shell formulations applied on numerical examples with adaptively refined FE meshes. Proc. *International Conference on Adaptive Modeling and Simulation, ADMOS 2003, Göteborg / Sweden*, CIMNE/Barcelona 2003.

[6] D. Fuentes, D. Littlefield, J.T. Oden and S. Prudhomme. Extensions of goal-oriented error estimation methods to simulations of highly-nonlinear response of shock-loaded elastomer-reinforced structures. *Comp. M. Appl. Mech. Eng.*, 195:4659-4680, 2006.

[7] S. Kizio and K. Schweizerhof. Efficient strategies for goal-oriented error estimation and mesh adaptation in structural dynamics. *Proc. CompDyn Conf.*, Greece, 2007.

[8] S. Kizio. *Adaptive Finite-Element-Algorithmen in der Strukturdynamik*, Diss. Universität Karlsruhe, 2008.

[9] J. Neumann, K. Schweizerhof. Computation of Single Eigenvalues and Eigenfunctions of Plate and Shell Structures using Adaptive FE-Methods. *Computational Mechanics*, 40 (1):111–126, (2007)

[10] K. Schweizerhof, J. Neumann and J. Riccius. *Adaptive Analysis of Plate and Shell Structures under Transient Loading*, in [11].

[11] E. Stein (Editor), E. Ramm, E. Rank, R. Rannacher, K. Schweizerhof, E. Stein, W. Wendland, G. Wittum, P. Wriggers, W. Wunderlich. *Error-controlled Adaptive Finite Elements in Solid Mechanics,* ISBN: 978-0-471-49650-2, Wiley, 2002.

[12] J. Hallquist. *LS-DYNA theoretical manual.* Livermore Software Techn. Corp., Livermore, California 2006.

[13] J. Hallquist. *LS-DYNA vs. 971 Keyword manual.* Livermore Software Techn. Corp., Livermore, California 2007.

On Adaptive Finite Element Analyses in Structural Dynamics

-

A specific view on practical engineering applications and engineering modeling

-

Karl Schweizerhof, Universität Karlsruhe/DYNAmore GmbH
Stephan Kizio, Universität Karlsruhe

7th European LS-DYNA Conference Salzburg, May 14-15, 2009

Outline

- Introduction
  - Adaptivity as supportive tool for practical applications of FEM
  - Requirements for practical applications of adaptive methods
- Spatially adaptive methods in structural dynamics
  - Goal-oriented h-adaptive schemes for semidiscrete FEM
  - Goal-oriented h-adaptivity for dynamic eigenvalue problem
- A look on industrial applications of adaptive schemes
  - Crash simulations, Metal forming
  - Open problems in industrial applications
- Using adaptivity to create/verify engineering models
- Conclusions

Motivation

Adaptive schemes are:

- well-known in the scientific community
- highly sophisticated

But:

- "rarely" used in practical applications WHY ?

Adaptive schemes should be (engineering point of view):

- supportive tools for modeling and discretization purposes

Demands:

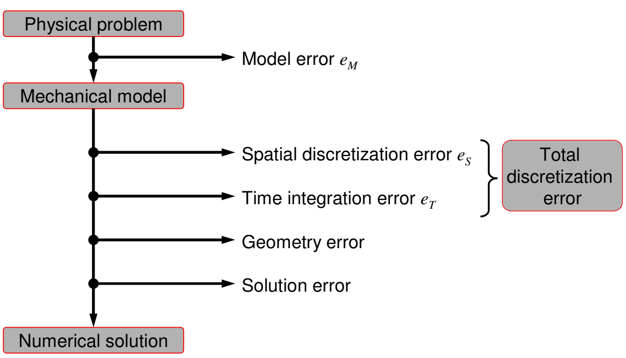
- minimal effort
- good indication of error in the quantity of interest
- hints for improvements of the model and discretization
  - e.g. use of different element modifications should be possible
  - assisting to create/verify simplified models
- provide further insight into the problem

Steps of simulation process

- Modeling
  - Beams – Shells – 3D-Continua
    - Details: e.g. which beam- or shell-formulation?
  - Constitutive model
  - Joints – w./wo. Friction (*Model adaptivity ?)*
- Finite element discretization
  - Element formulation – Time stepping scheme
  - Mesh density ➔ Adaptivity
  - Time step size ➔ Adaptivity
- Solution
- Verification of solution
  - Hints from Adaptivity

Considered here: Semidiscretization in structural dynamics

Errors in semidiscretization in structural dynamics



Splitting of total discretization error

$$\boldsymbol{e} = \boldsymbol{e}_S + \boldsymbol{e}_T = \underbrace{(\boldsymbol{u} - \boldsymbol{u}_h)}_{\boldsymbol{e}_S} + \underbrace{(\boldsymbol{u}_h - \boldsymbol{u}_{h,k})}_{\boldsymbol{e}_T}$$

With:

$\boldsymbol{u}$: exact solution
$\boldsymbol{u}_{h,k}$: numerical solution – applying spatial and temporal discretization
$\boldsymbol{u}_h$: exact solution of semidiscrete equation of motion
(no time integration error → not computable)

All other errors (model, geometry, solution) are neglected

Goal:

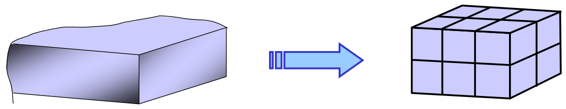
- Estimation and control of spatial discretization error in an arbitrary quantity of interest

Main focus here:

- Practical application of adaptive schemes. Possible simplifications ?
- Provide interpretations of adaptive schemes for engineering purposes

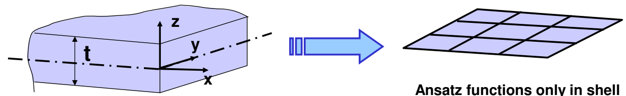
Modeling and Spatial FE Discretization

3D continuum



No specific assumptions for kinematics | **Ansatz functions in all directions identical**

Shell theory ⇨ kinematics $\varepsilon_{xx}, \varepsilon_{yy} = a \cdot z + b$ (linear in thickness direction)

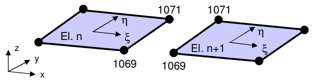


$\sigma_{zz}=0$
σ_{xz} ; σ_{yz} constant } ⇨ Reissner / Mindlin theory

Ansatz functions only in shell surface directions
in z-direction only loading !
(no z-stress)

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

FE Discretization and Spatial Error Estimation



Standard: C^0 - elements ⇨ **continuous in displacements/rotations**
not continuous in strains / stresses } between elements
⇨ **jumps in boundary terms ⇨ boundary error**
approximation in domain / in element ⇨ domain error

Classical error estimation:
(“a posteriori“ estimation)

$$\|e_s\| \le \underbrace{\sqrt{\sum_{elem}\left[\left\|C_{1k}\cdot h_k \cdot \mathrm{R}_1\right\|_{L2k}\right]^2}}_{\text{domain term}} + \underbrace{\sqrt{\sum_{elem}\left[\left\|C_{2k}\cdot \sqrt{h_k}\cdot \mathrm{R}_2\right\|_{L2\partial k}\right]^2}}_{\text{boundary term}}$$

Rem.: **independent of order of approximation**

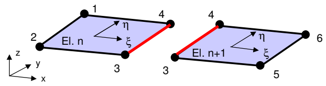
$\mathbf{R}_1$ = error of residuum (≡ satisfaction of DE in element)
$\mathbf{R}_2$ = stress jump at element borders
h_k = characteristic element size
C_{ik} = interpolation constant (can be computed from eigenvalue analysis for elements)

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in statics | Adaptive schemes in ind. applications | Conclusions

FE Discretization and Error estimation (contd.)

vectors $\mathbf{R}_i$ can be further subdivided e.g. for shells in normal-, membrane- or transversal shear error

Ex.: boundary term
stress jump :



Strains at boundary 3 -4 e.g.

in element n as: $\dfrac{\partial u}{\partial x} = \dfrac{\partial}{\partial x}\left(u = \sum_{1}^{4}\phi_i(\xi,\eta)u_i\right)$

in element n+1 as: $\dfrac{\partial u}{\partial x} = \dfrac{\partial}{\partial x}\left(u = \sum_{j=3}^{6}\phi_j(\xi,\eta)u_j\right)$

are **not identical**.

Reason: stress boundary terms are also dependent on quantities defined outside of common boundary

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in statics | Adaptive schemes in ind. applications | Conclusions

FE Discretization and Error Estimation (contd.)

Error estimation according to Zienkiewicz and Zhu:
„Superconvergent Patch Recovery“ 1992 IJNME

$$\|\mathbf{u}_h - \mathbf{u}_{ex}\|_{a,\Omega} = \int_\Omega (\boldsymbol{\sigma}_h - \boldsymbol{\sigma}_{ex})^T \mathbf{C}^{-1}(\boldsymbol{\sigma}_h - \boldsymbol{\sigma}_{ex})\,d\Omega \approx \int_\Omega (\boldsymbol{\sigma}_h - \boldsymbol{\sigma}^*)^T \mathbf{C}^{-1}(\boldsymbol{\sigma}_h - \boldsymbol{\sigma}^*)\,d\Omega$$

with: $\boldsymbol{\sigma}^* = \sum_{i=1}^{N_K}\mathbf{N}_i\sigma_i^*$ … improved Finite-Element stresses

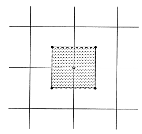
σ_i^* … improved Finite-Element nodal stresses

Approximation of stress components in patch

$\sigma_i^P(x,y) = \mathbf{P}(x,y)\mathbf{a}_i$ …stress components

$\mathbf{P}(x,y) = (1,x,y)$ …smoothing functions

$\mathbf{a}_i = (a_{i1}, a_{i2}, a_{i3})^T$ …coefficients



Rem.: **optimally used for low order test functions !**

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in statics | Adaptive schemes in ind. applications | Conclusions

FE discretization and Error Estimation (contd.)

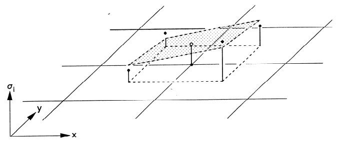
Error estimation according to Zienkiewicz and Zhu:
„Superconvergent Patch Recovery“ 1992 IJNME

Least square fit to determine $\mathbf{a}_i$: $F(\mathbf{a}_i) = \sum_{k=1}^{n}(\sigma_i^h(x_k,y_k) - \sigma_i^P(x_k,y_k))^2 \longrightarrow$ min

with: x_k, y_k ...coordinates of superconvergent stress points

σ_i^h ...stress components of FE-solution

n ...number of stress points in patch



Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in statics | Adaptive schemes in ind. applications | Conclusions

FE Discretization and Error Estimation (contd.)

Linear Statics:

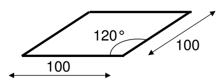
Global estimators of strain energy
Residual Concept (including stress jumps/residual terms) = classical concept
Babuska/Miller 1987; Johnson/Hansbo 1992
Patch - oriented estimators: *Zienkiewicz/Zhu 1992*

Nonlinear Statics : **Error estimation at end of converged step**
Rheinboldt 1985:
Working with linearized problem ➔ same error estimator as for linear problems
Restriction: Linearized problem must be close to nonlinear problem
➔ only small stepsize in nonlinear analyses allowed
Stein et. al. 1993:
Special treatment of stability problems e.g. vicinity of bifurcation points
Suttmeier/Rannacher 1996/97:
Plasticity - linearized dual problem ➔ very good results and efficient meshes for local quantities

Morley Plate

- A-posteriori error estimation
- Starting mesh: 1 finite element

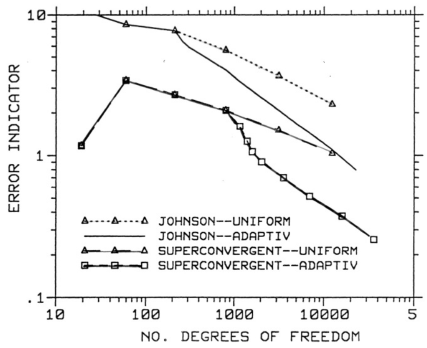
$E = 10^7$
$\nu = 0.3$
$p = 1$
$t = 1$



- The following results are obtained:
  - An adaptive analysis is more efficient than an uniform mesh
  - Comparison of error indicators:
    - The stress-jump indicator (Johnson / Hansbo) leads to an upper bound of the error
    - For both indicators the generated meshes are nearly identical

⇒ **Model problem: In boundary layer error is hardly decreasing.**

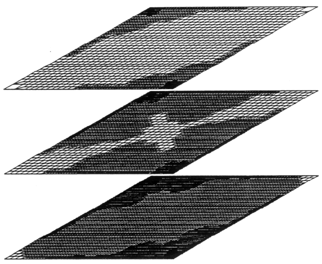
Morley Plate (contd.) : Convergence Diagram



Remarks:

- **Global error decreases**
- **Local error may be still larger (e.g. boundary layer)**

Morley Plate (contd.)



10. iteration
2322 elements

11. iteration
5316 elements

12. iteration
11796 elements

Meshes for different iteration states - superconvergent patch error indicator
Zienkiewicz / Zhu

Estimation of error in a quantity of interest

Main steps:

- ▶ Choose quantity of interest $Q(\mathbf{u})$
- ▶ Suitable error representation for the error $E(\mathbf{u},\mathbf{u}_h) = Q(\mathbf{u}) - Q(\mathbf{u}_h)$ applying Dual-Weighted-Residual- approach (DWR) see e.g. :
  - Eriksson & Johnson 1991
  - Eriksson, Estep, Hansbo & Johnson 1995
  - Rannacher & Becker 1996
  - Rannacher & Bangerth 2003
  - and others

Basic idea:

- ▶ Residual of equation of motion is weighted with dual solution (influence function)
- ▶ Choose initial conditions of dual problem w.r.t. the quantity of interest
- ▶ Numerical evaluation of error representation

Global vs. Local Error Estimation

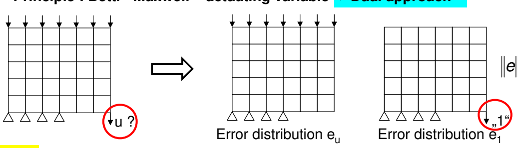
Global error

Sum of energy error over all elements / total energy
⇨ error can differ considerably dependent on location / but global behavior well captured

$$\|e\|_{Grob} = \left(\sum_i^{elem} \|e_{ui}\|^2 \right)^{\frac{1}{2}}$$

Local error

influence of corresponding error on a local quantity e.g. displacement/ force
Principle : Betti - Maxwell – actuating variable ⇨ Dual approach



$$\|e\| = \sqrt{\sum_{elem} \|e_{ui}\| \cdot \|e_{1i}\|}$$

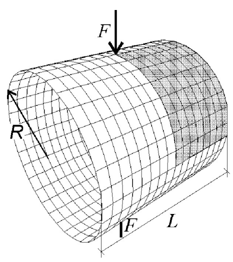
Error distribution e_u Error distribution e_1

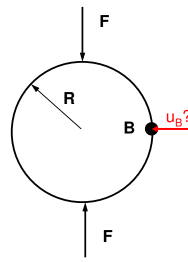
Rem.: Weighting of "global" error with local quantity

Rannacher / Becker / Suttmeier , Heidelberg 1997/98 ; Ramm / Cirak, Stuttgart 1998/99

Mesh development for different error estimation

Example: Pinched Cylinder



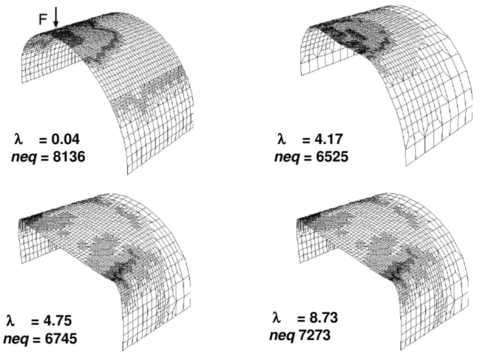


$F = 182.64$
$E = 3 \cdot 10^4$
$\nu = 0.3$
$L = 200$
$R = 100$
$t = 0.1$

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in nonlinear statics | Adaptive schemes in ind. applications | Conclusions

Mesh development for global error estimation

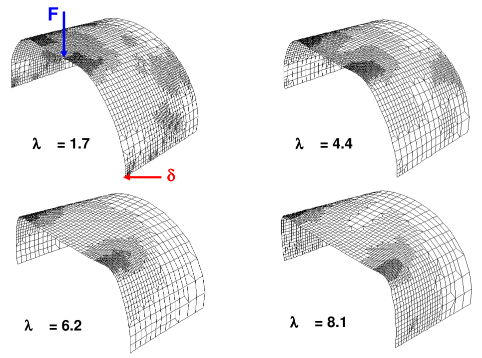


19

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in nonlinear statics | Adaptive schemes in ind. applications | Conclusions

Mesh adaptation with dual error estimation



20

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

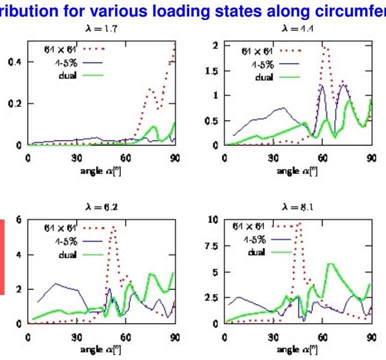
Motivation | Adaptive schemes in nonlinear statics | Adaptive schemes in ind. applications | Conclusions

Mesh adaptation with dual error estimation (contd.)

Absolute error distribution for various loading states along circumference

Comparison of

- **uniform mesh 61 × 61**
- **global error est. 4-5 %**
- **dual error estimation**

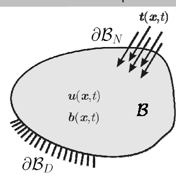


Dual error estimation → efficient mesh adaptation

21

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions



Equation of motion

$$\begin{aligned} div\,\boldsymbol{\sigma} + \rho\boldsymbol{b} - \rho\ddot{\boldsymbol{u}} &= 0 && \text{on } \mathcal{B} && 0 \le t \le T \\ \boldsymbol{\sigma}\boldsymbol{n} &= \boldsymbol{t} && \text{on } \partial\mathcal{B}_N && 0 \le t \le T \\ \boldsymbol{u} &= \boldsymbol{u}_D && \text{on } \partial\mathcal{B}_D && 0 \le t \le T \\ \boldsymbol{u}(t=0) &= \boldsymbol{u}_0 && && t = 0 \\ \dot{\boldsymbol{u}}(t=0) &= \dot{\boldsymbol{u}}_0 && && t = 0 \end{aligned}$$

Weak form of equation of motion (with damping)

$$\rho(\ddot{\boldsymbol{u}}, \boldsymbol{w}) + c_M\rho(\dot{\boldsymbol{u}}, \boldsymbol{w}) + c_K a(\dot{\boldsymbol{u}}, \boldsymbol{w}) + a(\boldsymbol{u}, \boldsymbol{w}) = \mathcal{F}_u(\boldsymbol{w}) \quad \forall \boldsymbol{w} \in W,\ 0 \le t \le T$$

With:

$$\begin{aligned} (\ddot{\boldsymbol{u}}, \boldsymbol{w}) &= \int_{\mathcal{B}} \ddot{\boldsymbol{u}} \cdot \boldsymbol{w}\, dV \\ a(\boldsymbol{u}; \boldsymbol{w}) &= \int_{\mathcal{B}} \boldsymbol{\sigma} : grad\boldsymbol{w}\, dV = \int_{\mathcal{B}} \boldsymbol{\sigma} : \partial\boldsymbol{\varepsilon}(\boldsymbol{w})\, dV \\ \mathcal{F}_u(\boldsymbol{w}) &= \int_{\mathcal{B}} \rho\boldsymbol{b} \cdot \boldsymbol{w}\, dV + \int_{\partial\mathcal{B}_N} \boldsymbol{t} \cdot \boldsymbol{w}\, dA \end{aligned}$$

22

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Physical interpretation of weak formulation

Interpretation of the testfunction $\boldsymbol{w}$

as

virtual displacement	virtual velocity
$\boldsymbol{w} = \partial\boldsymbol{u}$	$\boldsymbol{w} = \partial\dot{\boldsymbol{u}}$

yields

Principle of virtual work	Principle of virtual power
$\partial\Pi = 0$	$\partial P = 0$

Both principles yield different reciprocity theorems

23

Institut für Mechanik | DYNAmore | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Spatial discretization

Assumption: Only spatial discretization → no time integration error

$$\rho(\ddot{\boldsymbol{u}}_h, \boldsymbol{w}_h) + c_M\rho(\dot{\boldsymbol{u}}_h, \boldsymbol{w}_h) + c_K a(\dot{\boldsymbol{u}}_h, \boldsymbol{w}_h) + a(\boldsymbol{u}_h, \boldsymbol{w}_h) = \mathcal{F}_u(\boldsymbol{w}_h) \quad \forall \boldsymbol{w}_h \in W^h,\ 0 \le t \le T$$

Residual

$$\mathcal{R}_u(\boldsymbol{w}) = \mathcal{F}_u(\boldsymbol{w}) - (\rho_0(\ddot{\boldsymbol{u}}_h, \boldsymbol{w}) + c_M\rho(\dot{\boldsymbol{u}}_h, \boldsymbol{w}) + c_K a(\dot{\boldsymbol{u}}_h, \boldsymbol{w}) + a(\boldsymbol{u}_h, \boldsymbol{w}))$$

$$\mathcal{R}_u(\boldsymbol{w}) = \rho_0(\ddot{\boldsymbol{e}}_S, \boldsymbol{w}) + c_M\rho(\dot{\boldsymbol{e}}_S, \boldsymbol{w}) + c_K a(\dot{\boldsymbol{e}}_S, \boldsymbol{w}) + a(\boldsymbol{e}_S, \boldsymbol{w}) \quad \forall \boldsymbol{w} \in W,\ 0 \le t \le T$$

← Differential equation of discretization error

Galerkin orthogonality: $\mathcal{R}_u(\boldsymbol{w}_h) = 0 \quad \forall \boldsymbol{w}_h \in W^h,\ 0 \le t \le T$

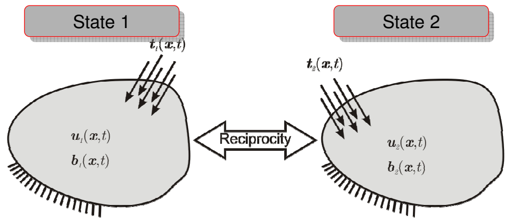
Summary:

- Spatial discretization error is a solution of the equation of motion with residual as loading function
- Spatial discretization error is an elastodynamic state for which we can apply elastodynamic reciprocity theorems

24

Reciprocity in elastodynamics

„Reciprocity is a good thing. Something is given and something else, equally or more valuable, is returned. [Achenbach, 2003]



- Reciprocity theorems relate two admissible elastodynamic states of an elastic body
- Two reciprocity theorems formulated in the time domain based on:
  - Principle of virtual work (Graffi-Theorem)
  - Principle of virtual power

Reciprocity in elastodynamics (2)

Reciprocity in the time domain based on principle of virtual work

Graffi (1946): Given two elastodynamic states $\boldsymbol{u}_1(x,t)$ and $\boldsymbol{u}_2(x,t)$ of an elastic body, for each time t_n the following reciprocity theorem h

$$\int_{\partial\mathcal{B}} \boldsymbol{t}_1(\boldsymbol{x},t_n) * \boldsymbol{u}_2(\boldsymbol{x},t_n)dA + \int_{\mathcal{B}} \rho\boldsymbol{b}_1(\boldsymbol{x},t_n) * \boldsymbol{u}_2(\boldsymbol{x},t_n)dV$$
$$+ \int_{\mathcal{B}} \rho\boldsymbol{u}_1(\boldsymbol{x},0)\dot{\boldsymbol{u}}_2(\boldsymbol{x},t_n) + \rho\dot{\boldsymbol{u}}_1(\boldsymbol{x},0)\boldsymbol{u}_2(\boldsymbol{x},t_n)dV =$$
$$\int_{\partial\mathcal{B}} \boldsymbol{t}_2(\boldsymbol{x},t_n) * \boldsymbol{u}_1(\boldsymbol{x},t_n)dA + \int_{\mathcal{B}} \rho\boldsymbol{b}_2(\boldsymbol{x},t_n) * \boldsymbol{u}_1(\boldsymbol{x},t_n)dV$$
$$+ \int_{\mathcal{B}} \rho\boldsymbol{u}_2(\boldsymbol{x},0)\dot{\boldsymbol{u}}_1(\boldsymbol{x},t_n) + \rho\dot{\boldsymbol{u}}_2(\boldsymbol{x},0)\boldsymbol{u}_1(\boldsymbol{x},t_n)dV \qquad \forall t_n > 0$$

With the convolution: $\boldsymbol{a}(t_n) * \boldsymbol{b}(t_n) = \int_0^{t_n} \boldsymbol{a}(\tau)\cdot\boldsymbol{b}(t_n-\tau)d\tau$

Note: No damping for brevity reasons – damping can be added fairly simply

Reciprocity in elastodynamics (3)

Reciprocity in the time domain based on principle of virtual power

Given two elastodynamic states $\boldsymbol{u}_1(x,t)$, $\boldsymbol{v}_1(x,t)$ and $\boldsymbol{u}_2(x,t)$, $\boldsymbol{v}_2(x,t)$ of an elastic body, for each time t_n the following reciprocity theorem holds:

$$\int_{\partial\mathcal{B}} \boldsymbol{t}_1(\boldsymbol{x},t_n) * \boldsymbol{v}_2(\boldsymbol{x},t_n)dA + \int_{\mathcal{B}} \rho\boldsymbol{b}_1(\boldsymbol{x},t_n) * \boldsymbol{v}_2(\boldsymbol{x},t_n)dV$$
$$+ \int_{\mathcal{B}} \rho\boldsymbol{v}_1(\boldsymbol{x},0)\boldsymbol{v}_2(\boldsymbol{x},t_n) + \boldsymbol{\sigma}_1(\boldsymbol{x},0) : \boldsymbol{\varepsilon}_2(\boldsymbol{x},t_n)dV =$$
$$\int_{\partial\mathcal{B}} \boldsymbol{t}_2(\boldsymbol{x},t_n) * \boldsymbol{v}_1(\boldsymbol{x},t_n)dA + \int_{\mathcal{B}} \rho\boldsymbol{b}_2(\boldsymbol{x},t_n) * \boldsymbol{v}_1(\boldsymbol{x},t_n)dV$$
$$+ \int_{\mathcal{B}} \rho\boldsymbol{v}_2(\boldsymbol{x},0)\boldsymbol{v}_1(\boldsymbol{x},t_n) + \boldsymbol{\sigma}_2(\boldsymbol{x},0) : \boldsymbol{\varepsilon}_1(\boldsymbol{x},t_n)dV \qquad \forall t_n > 0$$

Note: Theorem can be derived in same fashion as Graffi-Theorem

Reciprocity theorems are independent of the discretization

Reciprocity and error estimation

General idea:

- State 1: spatial discretization error $\boldsymbol{e}_S$ ➔ Residual is loading function
- State 2: auxiliary (dual) problem with solution z ➔ homogeneous problem

Two identities:

1. Based on principle of **virtual displacement** – displacement z of dual problem as weighting function of residual – via Graffi-theorem:

$$[\rho(\dot{\boldsymbol{e}}_S,\boldsymbol{z}) - \rho(\boldsymbol{e}_S,\dot{\boldsymbol{z}}) + c_M\rho(\boldsymbol{e}_S,\boldsymbol{z}) + c_K a(\boldsymbol{e}_S,\boldsymbol{z})]_0 + \int_0^{t_n}\mathcal{R}_u(\boldsymbol{z})dt$$
$$= [\rho(\boldsymbol{z},\dot{\boldsymbol{e}}_S) - \rho(\dot{\boldsymbol{z}},\boldsymbol{e}_S) + c_M\rho(\boldsymbol{z},\boldsymbol{e}_S) + c_K a(\boldsymbol{z},\boldsymbol{e}_S)]^{t_n}$$

   Used for definition of initial conditions of dual problem

2. Based on principle of **virtual velocities** – velocities $\dot{z}$ of dual problem as weighting function of residual:

$$[\rho(\dot{\boldsymbol{e}}_S,\dot{\boldsymbol{z}}) + a(\boldsymbol{e}_S,\boldsymbol{z})]_0 + \int_0^{t_n}\mathcal{R}_u(\dot{\boldsymbol{z}})dt$$
$$= [\rho(\dot{\boldsymbol{z}},\dot{\boldsymbol{e}}_S) + a(\boldsymbol{z},\boldsymbol{e}_S)]^{t_n}$$

   Used for definition of initial conditions of dual problem

Weak formulation of dual problem

From definition of convolution:

- **Backward problem in time**

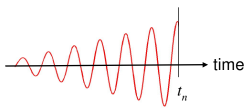
$$\rho(\ddot{\boldsymbol{z}},\boldsymbol{w}) - c_M\rho(\dot{\boldsymbol{z}},\boldsymbol{w}) - c_K a(\dot{\boldsymbol{z}},\boldsymbol{w}) + a(\boldsymbol{z},\boldsymbol{w}) = 0$$
$$\forall \boldsymbol{w} \in W,\ 0 \le t \le t_n$$

- **Initial conditions at "end" time t_n**

Features of dual problem:

- Dual problem describes the spatial and temporal transport
- Negative damping terms: Only limited temporal transport for damped problems („loss of memory"):

Temporal evolution of dual solution for damped problem:



Quantity of interest

Target functional: $Q(\boldsymbol{u})|^{t_n}$

Error measure: $E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = Q(\boldsymbol{u})|^{t_n} - Q(\boldsymbol{u}_h)|^{t_n}$

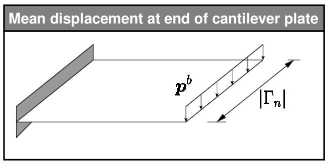
Example: Mean error of the displacements in subdomain at time t_n

$$Q(\boldsymbol{u})|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n}\boldsymbol{p}^b\cdot\boldsymbol{u}ds|^{t_n}$$

with $|\boldsymbol{p}^b| = 1$

$$E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n}\boldsymbol{p}^b\cdot\underbrace{(\boldsymbol{u}-\boldsymbol{u}_h)}_{\boldsymbol{e}_S}ds|^{t_n}$$

Mean displacement at end of cantilever plate



Initial conditions of dual problem define the quantity of interest

Error representations for the quantity of interest

Error representation based on principle of virtual work

Initial conditions of dual problem are chosen such that:

$$E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = \rho(\boldsymbol{z},\dot{\boldsymbol{e}}_S)|^{t_n} - \rho(\dot{\boldsymbol{z}},\boldsymbol{e}_S)|^{t_n} + c_M\rho(\boldsymbol{z},\boldsymbol{e}_S)|^{t_n} + c_K a(\boldsymbol{z},\boldsymbol{e}_S)|^{t_n}$$

Example: Mean error of the displacements in subdomain at time t_n

Error measure: $E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n} \boldsymbol{p}^b \cdot \boldsymbol{e}_S \, ds|^{t_n}$

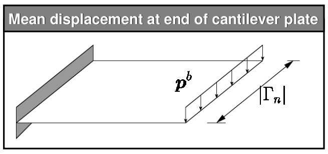
Initial conditions

$\boldsymbol{z}(t_n) = \boldsymbol{0}$ (chosen)

$-\rho_0(\dot{\boldsymbol{z}},\boldsymbol{e}_S)|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n} \boldsymbol{p}^b \cdot \boldsymbol{e}_S \, ds|^{t_n}$

Impulse load at time t_n ➔ $\dot{z}(t_n)$

➔ **Backward analysis**



Error representations for the quantity of interest (2)

Error representation based on principle of virtual power

Initial conditions of dual problem are chosen such that:

$$E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = \rho(\dot{\boldsymbol{e}}_S,\dot{\boldsymbol{z}})|^{t_n} + a(\boldsymbol{e}_S,\boldsymbol{z})|^{t_n}$$

Example: Mean error of the displacements in subdomain at time t_n

Error measure: $E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n} \boldsymbol{p}^b \cdot \boldsymbol{e}_S \, ds|^{t_n}$

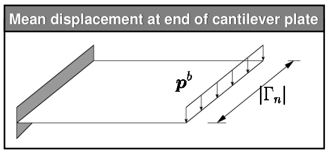
Initial conditions

$\dot{\boldsymbol{z}}(t_n) = \boldsymbol{0}$ (chosen)

$a(\boldsymbol{z},\boldsymbol{e}_S)|^{t_n} = \frac{1}{|\Gamma_n|}\int_{\Gamma_n} \boldsymbol{p}^b \cdot \boldsymbol{e}_S \, ds|^{t_n}$

Quasi-static problem at time t_n ➔ $z(t_n)$

➔ **Backward analysis**



Classification of solutions of equation of motion

Equation of motion

Depending on: Loading, boundary conditions etc.

Wave propagation

- Examples: Impact, Shockwaves
- Short time dynamics
- High frequencies
- Transport of energy is dominant
- Discretization must capture the transport

Vibration

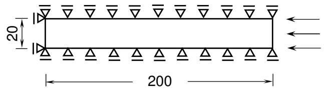
- Example: harmonic oscillation
- Long time dynamics
- Lower frequencies
- Transport of energy is of minor interest
- Classic solution: Modal superposition

Remark: Adaptive scheme should take into account physical nature of the considered problem ⟶ **problem oriented simplification ?**

Example: Adaptive computation of wave propagation

Thin rectangular slab subjected to half-sinusoidal impulse

Problem

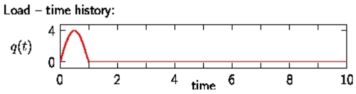


Material: $\rho = 10.0$, $E = 10000.0$, $\nu = 0.3$, $t = 1.0$

Error bounds: $\epsilon_{lower} = 0.0$, $\epsilon_{upper} = 3\%$, $\epsilon_{presc} = 2.5\%$

From pre-simulation: $\|\boldsymbol{u}\|_{max,pre} = 0.89$

Load – time history:



Adaptive scheme

- Error estimation with standard energynorm estimators (global)
- Hierarchical mesh refinement and coarsening

Wave propagation: FE-meshes at different time intervals

t ∈ (0.2, 0.5]

t ∈ (1.2, 1.5]

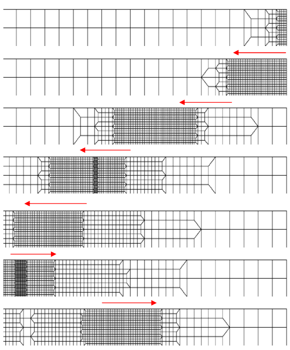
t ∈ (3.7, 4.1]

t ∈ (4.2, 4.5]

t ∈ (5.0, 5.4]

t ∈ (5.7, 6.6]

t ∈ (9.6, 9.9]

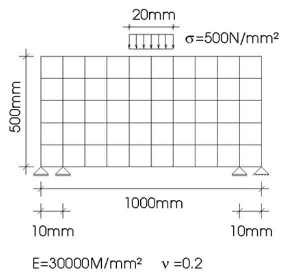


- Refined regions propagate with the wave
- Mesh refinement only in restricted area

Example: Plane pressure wave in concrete specimen

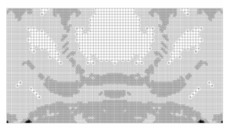
Simulation of an explosive charge on a concrete specimen

Problem



E=30000M/mm² ν =0.2

Mesh evolution



Time span: 0 sec. – 0,002 sec

Practical problems of wave propagation simulations

- Dispersion due to spatial discretization
- Reflections at boarders of fine to coarse meshes
- Error due to transfer between different meshes
- Demands for transfer of data:
  - Global energy preservation?
  - Preservation of local behavior of wave
- Complex geometries ➔ often fairly uniform meshes needed

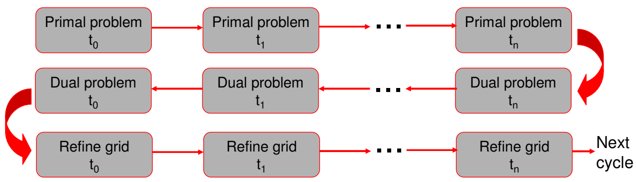
But: reduction of effort with DRW approach (Bangerth & Rannacher 2001)

37

Goal-oriented adaptivity for wave propagation problems

See: Rannacher/Bangerth (1999), Fuentes, Oden, Littlefield, Prudhomme(2005)

- Temporal evolution of error is of high practical interest
  - Full dual problem is needed
- Adaptive scheme [from Rannacher/Bangerth (1999)]:



Advantage:

- Complete information of dual problem used for adaptation
  ⟶ Full error information

Drawbacks:

- Very high numerical effort
- Large memory requirements
- Transfer/estimation with different refined meshes ?

38

Goal oriented error estimation for **vibration problems**

Modal decomposition of spatial discretization error

Exact solution: $u(x,t) = \sum_{i=1}^{\infty} U_i(x)\cdot f_i(t)$

Discrete solution: $u_h(x,t) = \sum_{i=1}^{n_{dof}} U_i^h(x)\cdot f_i^h(t)$

Discretization error: $$e_S(x,t) = \sum_{i=1}^{n_{dof}} \Big(\underbrace{(U_i - U_i^h)}_{E_i(x)}\cdot f_i^h(t) + U_i\cdot \underbrace{(f_i(t)-f_i^h(t))}_{e_\varphi(t)}\Big) + h.o.t.$$

Spatial discretization error consists of:

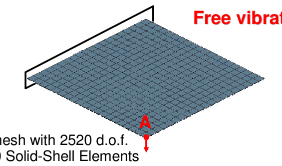
- Error in spatial approximation of natural modes $\sum_{i=0}^{n_{dof}} (E_i(x)\cdot f_i^h(t))$
- Error in approximation of natural frequencies - phase error $\sum_{i=0}^{n_{dof}} (U_i\cdot e_\varphi(t))$
- Truncation of higher modes (h.o.t.) ➔ usually negligible

Question: Which part of the error could be neglected for practical purposes?

39

Example for phase error due to spatial discretization

Free vibration of a plate in 3. natural frequency



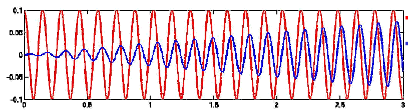
h-mesh with 2520 d.o.f.
400 Solid-Shell Elements

Reference solution:
Mesh with 6400 Elements and 38800 d.o.f.

Error in 3. natural frequency: 0.6%

Free vibration in 3. eigenmode considered

Quantity of interest: Vertical displacement at point A



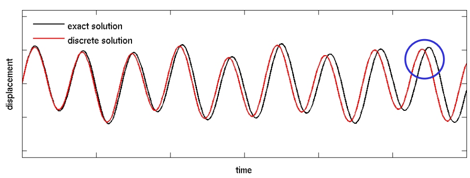
Note: **Phase error can have order of magnitude of the exact displacement**
For practical purposes: **Exact temporal solution of minor interest**
More important: **Maximum displacements, stresses etc.**

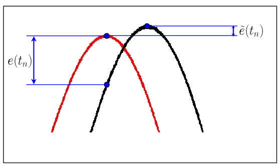
Proposition: Phase error should be neglected

40

Error measure without phase error

Characteristic evolution of exact and discrete solution for vibration problem





$e(t_n)$: total error at time t_n
comparison of different states in discrete and exact solution ($\dot u_h = 0$ but $\dot u \neq 0$)

➔ estimation with full backward problem

$\tilde e(t_n)$:
comparison of same states in discrete and exact solution ($\dot u_h = 0$ and $\dot u = 0$)

➔ estimation with state variables at t_n

41

Consequences of neglecting phase error

Modified error measure: $\tilde e_S(x,t) = \sum_{i=1}^{n_{dof}} E_i(x)\cdot f_i^h(t)$

- No phase shift between exact and numerical solution $f_i^h(t) = f_i(t)$
- Temporal evolution of spatial discretization error assumed to be known
- Current state is „accepted“ ➔ only state variables at current time necessary for error estimation
- **Proposed approach:** Use terms at end time t_n for error estimation
  - Only „spatial information“ of dual problem is necessary
  - Error representation based on **velocities of dual problem** is used

Error representation: $E(u,u_h) = [\rho_0(\dot{\tilde e}_S, \dot z) + a(e_S, z)]^{t_n}$

- ***Goal:*** Deriving simple error indicator to judge the current mesh w.r.t. quantity $Q(u)$

42

A simple error indicator

Subtracting Galerkin orthogonality & Application of Cauchy-Schwarz inequality yields:

$$|E(\boldsymbol{u},\boldsymbol{u}_h)|^{t_n} \leq [\rho||\dot{\boldsymbol{e}}_S||_{L_2}\cdot||\dot{\boldsymbol{e}}_Z||_{L_2} + ||\boldsymbol{e}_S||_a\cdot||\boldsymbol{e}_Z||_a]^{t_n}$$

Estimation of energynorm $||\cdot||_a$ by Zienkiewicz-Zhu error indicator (1992)

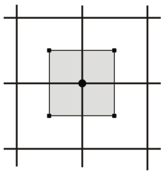
$$||\boldsymbol{e}_S||_a = \int_{\mathcal{B}} (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) : \mathbb{C}^{-1} : (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h)dV$$

Basic idea:

- Postprocessing of stresses with
- Superconvergent patch recovery (SPR)

Advantages:

- Implementation simple
- Simple to use with different element formulations (only stresses are needed for error estimation)



43

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

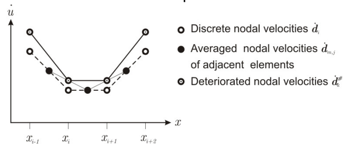
A simple error indicator (2)

Estimation of L2-norm $||\cdot||_{L2}$: Estimator with deteriorated velocities *(Riccius/Schweizerhof [1998])*

$$||\dot{\boldsymbol{e}}_S||^2_{L_2} \approx c^2||\dot{\boldsymbol{u}}^\# - \dot{\boldsymbol{u}}_h||_{L_2} = c^2\int_{\mathcal{B}} (\dot{\boldsymbol{u}}^\# - \dot{\boldsymbol{u}}_h)^2 dV$$

with $c = 0.365$

Illustration for 1-D-problem:



Basic ideas:

- No superconvergent points for velocities
- Generate field of deteriorated velocities (= velocities on coarser mesh ➔ factor c from similar problems)

Advantage:

- Numerical effort fairly small

44

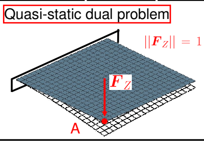
Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

A simple error indicator (3)

Example for error indicator: Estimation of vertical displacement at particular point A

1. Error estimation for primal problem: $||\boldsymbol{e}_S||_a$
2. Solve discrete dual problem: $\boldsymbol{K}\boldsymbol{d}_Z = \boldsymbol{F}_Z \rightarrow \boldsymbol{z}_h$
3. Error estimation for dual problem: $||\boldsymbol{e}_Z||_a$
4. Multiplication: $|E_{est}|^{t_n} = ||\boldsymbol{e}_S||_a \cdot ||\boldsymbol{e}_Z||_a$

Quasi-static dual problem



Advantages of error estimation by simplified approach:

- No backward integration of dual problem
- Dual problem must only be computed once and if mesh is changed
- Suitable basis for mesh adaptation at current time t_n
- Efficient use of important features of the dual problem

Main disadvantage:

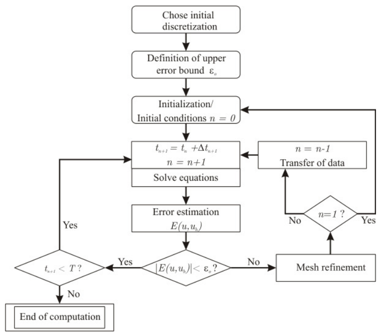
- No control of total discretization error

45

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Goal-oriented h-adaptive schemes

Strategy I: Single pass with sequential mesh adaptation and transfer of data



Advantages:

- Only one computation of whole problem necessary

Shortcomings:

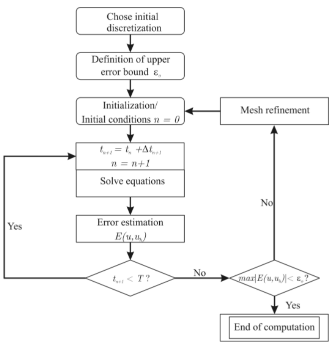
- Additional error due to transfer of data
- Resulting mesh only reflects the states at times of mesh modifications

46

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Goal-oriented h-adaptive schemes (2)

Strategy II: Multi pass with mesh adaptation at final step without transfer of data



Advantages:

- Error at states of all times captured for mesh adaptation
- No transfer error
- More robust than strategy I
- Very attractive for highly nonlinear analysis and robust

Shortcoming:

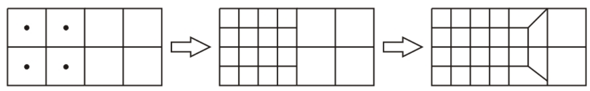
- Several re-computations of whole problem necessary

47

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Mesh refinement scheme for bilinear shell elements

Hierarchical mesh refinement with transition elements



Mark elements for refinement | Refine marked elements | Introduce transition elements to enforce compatibility

Advantage:

- Only small difference in subsequent meshes
- Clear hierarchy of meshes ➔ transfer of data very simple (coarse to fine)

Disadvantage:

- Transition elements ➔ badly shaped elements

48

Consistent transfer procedure *(Ortiz/Radovitzky 1999)*

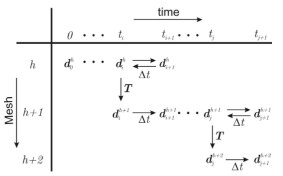
For Newmark scheme: (Note: CG version)

$$d_{n+1} = T d_n + \Delta t T \dot{d}_n + \Delta t^2 \left[(\tfrac{1}{2} - \beta) T \ddot{d}_n + \beta \ddot{d}_{n+1} \right]$$
$$\dot{d}_{n+1} = T \dot{d}_n + \Delta t \left[(1-\gamma) T \ddot{d}_n + \gamma \ddot{d}_{n+1} \right]$$

Transfer operator: $T = M_{n+1}^{-1} \cdot \int_{\mathcal{B}} \rho N_{n+1}^T \cdot N_n dV$

For hierarchical refinement: $N_n = N_{n+1} \cdot C$ — C = Constraint matrix

$$T = \underbrace{M_{n+1}^{-1} \cdot \int_{\mathcal{B}_0} \rho_0 N_{n+1}^T \cdot N_{n+1} dV}_{M_{n+1}} \cdot C = C$$



Steps of transfer procedure:

- Geom. interpolation of solution at t_n
- Compute solution at t_{n+1} with time integration scheme

49

Consistent transfer procedure (2)

Some demands for general transfer operators (Rashid 2002):

- Transfer should generate sufficiently smooth solution
- Transfer should pertain local character of solution (e.g. wave propagation)
- Self consistency
- Global energy preservation vs. Local momentum preservation?
- Application possible also to modified element formulations:
  - Assumed strain formulations
  - Enhanced strains
  - Underintegrated elements

Practical questions:

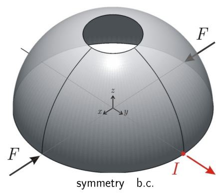
- Error due to transfer?
- Which features of transfer operator are important for the problem at hand ?

Remark: For practical purposes the transfer operator should be designed w.r.t. the physical nature of the computed problem

50

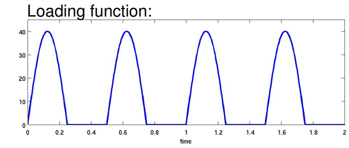
Numerical example

Hemisphere: Mesh adaptation due to displacement error at point *I*

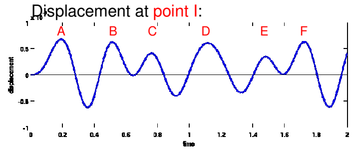


radius of hemisphere:	R	$= 10$
thickness	t	$= 0.04$
radius of hole:	r	$= 3$
modulus of elasticity:	E	$= 6.8 \cdot 10^7$
Poisson ratio:	ν	$= 0.3$
density:	ρ_0	$= 5$
Rayleigh damping:	c_m	$= 0.0003$
	c_k	$= 0.0001$
Time stepping: CG version	Newmark scheme with $\Delta t = 0.005$	

Loading function:



Displacement at point I:



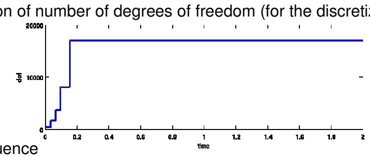
Spatial discretization:
Bilinear Solid-Shell-Elements ANS3DEAS

51

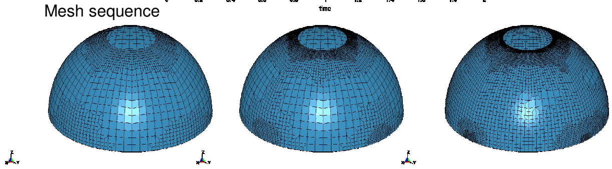
Numerical example (2)

Hemisphere: Strategy 1 – mesh adaptation with transfer of data

Evolution of number of degrees of freedom (for the discretized quarter)



Mesh sequence



3. Mesh 4. Mesh 5. Mesh

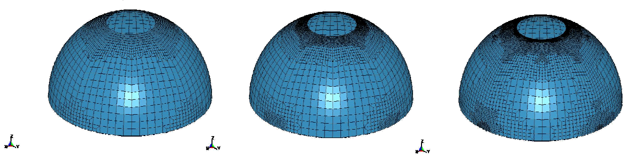
52

Numerical example (3)

Hemisphere: Strategy 2 – mesh adaptation without transfer of data

- 5 computations of whole problem necessary to reach desired accuracy

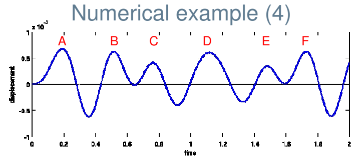
Mesh sequence



3. Mesh 4. Mesh 5. Mesh

53

Numerical example (4)



Relative errors at maximum amplitudes – comparison with uniform mesh

Uniform mesh as reference solution with ndof = 99500

Time	Uniform mesh ndof =24960	Strategy I $ndof_{max}$ =16972	Strategy II ndof =15516
A	0.3 %	0.8 %	0.8 %
B	0.4 %	0.2 %	0.8 %
C	2.1 %	0.4 %	1.0 %
D	0.4 %	0.2 %	0.7 %
E	4.2 %	0.2 %	0.4 %
F	0.9 %	0.3 %	0.9 %

54

Adaptive schemes in semidiscretization - Conclusions

Simplification of error indicator

- Simplification mandatory for large size, practical applications
- Use of standard energynorm indicator possible

Adaptive schemes

- Adaptation yields efficient spatial discretizations compared to uniform refinement
- Scheme without transfer of data more robust (in complex cases)

Limitations

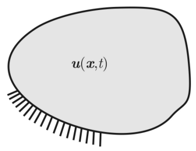
- Restriction to vibration problems
- No control of total error

Alternative: Goal-oriented adaptivity for eigenfunctions for Modal analysis

Basic idea:

- Split quantity of interest into modal parts: $Q(\boldsymbol{u}(\boldsymbol{x},t)) = \sum_{i=1}^{\infty} Q(\bar{\boldsymbol{u}}_i(\boldsymbol{x})) \cdot f_i(t)$
- Mesh adaptation for quantity of interest in single eigenmodes
  - Rannacher & Heuveline 2001
  - Oden, Prudhomme, Westerman & Bass 2003

Dynamic eigenvalue problem



Homogeneous equation of motion

$$div\boldsymbol{\sigma}(\boldsymbol{u}(\boldsymbol{x},t)) - \rho\ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \boldsymbol{0}$$

Continuous eigenvalue problem

$$div\boldsymbol{\sigma}(\bar{\boldsymbol{u}}) + \rho\lambda\bar{\boldsymbol{u}} = 0$$

Natural frequencies

$$\lambda = \omega^2 = 2\pi f$$

Dynamic eigenvalue problem – weak formulation

Generalized eigenvalue problem and normalization of modes:

$$a(\bar{\boldsymbol{u}},\boldsymbol{w}) = \lambda\rho(\bar{\boldsymbol{u}},\boldsymbol{w}) \qquad \forall \boldsymbol{w} \in W$$
$$\rho(\bar{\boldsymbol{u}},\bar{\boldsymbol{u}}) = 1$$

Normalization necessary to define unique solution

Compact notation

Eigenpair: $\boldsymbol{U} = \{\bar{\boldsymbol{u}},\lambda\} \in \mathcal{V} = W \times \mathbb{R}$

Testpair: $\boldsymbol{W} = \{\boldsymbol{w},\pi\} \in \mathcal{V} = W \times \mathbb{R}$

With: $A(\boldsymbol{U};\boldsymbol{W}) = \lambda\rho(\bar{\boldsymbol{u}},\boldsymbol{w}) - a(\bar{\boldsymbol{u}},\boldsymbol{w}) + \pi[\rho(\bar{\boldsymbol{u}},\bar{\boldsymbol{u}}) - 1]$

Eigenvalue problem & normalization:

$$A(\boldsymbol{U};\boldsymbol{W}) = 0 \quad \forall \boldsymbol{W} \in \mathcal{V}$$

Dynamic eigenvalue problem – discretization

Discretization: $A(\boldsymbol{U}_h;\boldsymbol{W}_h) = 0 \quad \forall \boldsymbol{W}_h \in \mathcal{V}^h = W^h \times \mathbb{R}$

Matrix representation: $(\boldsymbol{K} - \lambda_h \boldsymbol{M})\boldsymbol{d}_u = \boldsymbol{0}$

$$|\boldsymbol{d}_u^T \boldsymbol{M} \boldsymbol{d}_u| = 1$$

Dual problem for linear target functionals

Minimization problem (Rannacher & Becker 1996; Oden, Prudhomme et al. 2003):

with $Q(\boldsymbol{U})$ as an arbitrary functional „optimal control" leads to ➔ find $\boldsymbol{U}$ such that:

$$Q(\boldsymbol{U}) = \inf_{\boldsymbol{V} \in M} (Q(\boldsymbol{V})) \quad \text{with} \quad M = \{\boldsymbol{V} \in \mathcal{V}; A(\boldsymbol{V};\boldsymbol{W}) = 0 \ \ \forall \boldsymbol{W} \in \mathcal{V}\}$$

For single eigenvalues

Find saddle point $(\boldsymbol{U},\boldsymbol{Z}) \in \mathcal{V} \times \mathcal{V}$ with $\boldsymbol{Z} = \{\boldsymbol{z},\mu\} \in \mathcal{V}$

of the Lagrangian: $\mathcal{L}((\boldsymbol{U},\boldsymbol{Z})) = Q(\boldsymbol{U}) - A(\boldsymbol{U};\boldsymbol{Z})$

Dual problem for linear target functionals (2)

The saddle point problem yields:

$$A(\boldsymbol{U};\boldsymbol{W}) = 0 \qquad \forall \boldsymbol{W} \in \mathcal{V}$$ ← Eigenvalue problem

$$A'(\boldsymbol{U};\boldsymbol{V},\boldsymbol{Z}) = Q(\boldsymbol{V}) \qquad \forall \boldsymbol{V} \in \mathcal{V}$$ ← dual problem

Final dual problem for error estimation for linear target functionals:

$$-a(\boldsymbol{v},\boldsymbol{z}) + \lambda\rho(\boldsymbol{v},\boldsymbol{z}) = Q(\boldsymbol{V}) - Q(\boldsymbol{U})\rho(\bar{\boldsymbol{u}},\boldsymbol{v}) \qquad \forall \boldsymbol{V} \in \mathcal{V}$$

orthogonality condition: $\rho(\bar{\boldsymbol{u}},\boldsymbol{z}) = 0$

Error representation and error indicator

Error of primal problem: $\boldsymbol{E}_u = \boldsymbol{U} - \boldsymbol{U}_h = \{\bar{\boldsymbol{u}},\lambda\} - \{\bar{\boldsymbol{u}}_h,\lambda_h\} = \{\boldsymbol{e}_u, e_\lambda\}$

Error of dual problem: $\boldsymbol{E}_z = \boldsymbol{Z} - \boldsymbol{Z}_h = \{\boldsymbol{z},\mu\} - \{\boldsymbol{z}_h,\mu_h\} = \{\boldsymbol{e}_z, e_\mu\}$

Error representation (see e.g. Oden, Prudhomme et al. 2003):

$$E(\boldsymbol{U},\boldsymbol{U}_h) = \mathcal{R}_u(\boldsymbol{E}_z) + \underbrace{e_\lambda(\boldsymbol{e}_u,\boldsymbol{z}) + \frac{1}{2}\mu\rho(\boldsymbol{e}_u,\boldsymbol{e}_u)}_{\Delta_A}$$

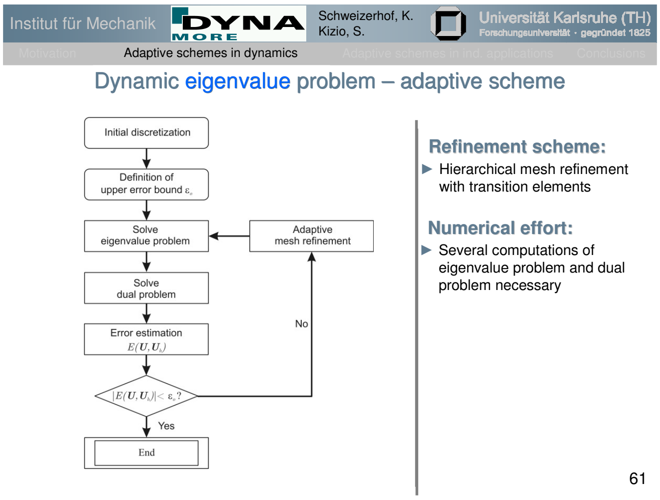
Small – higher order

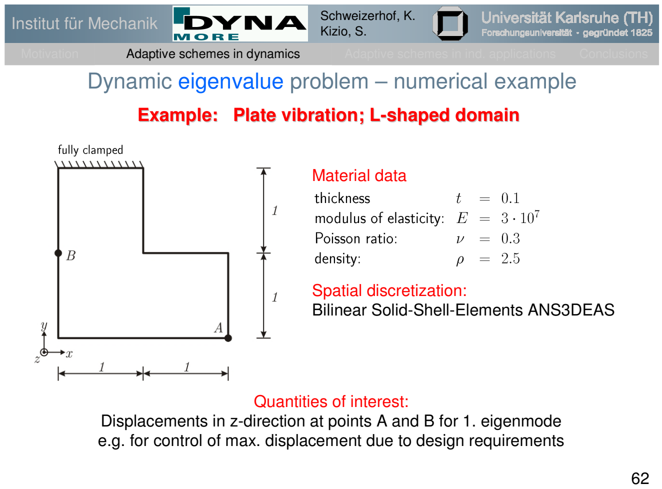
$$E(\boldsymbol{U},\boldsymbol{U}_h) \approx \mathcal{R}_u(\boldsymbol{E}_z)$$

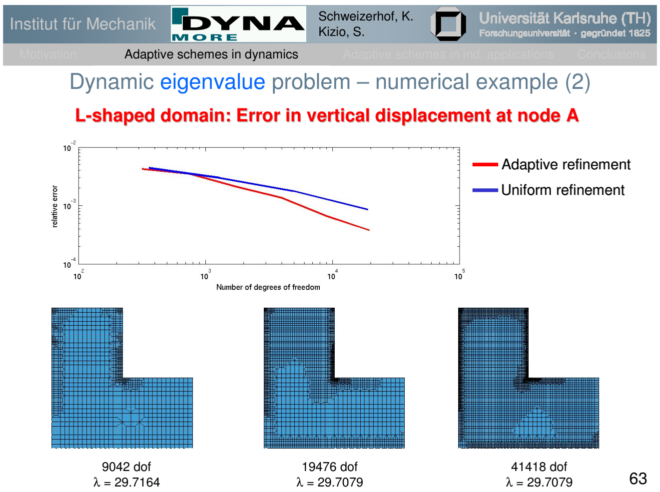
Error can be expressed by dual weighted residual

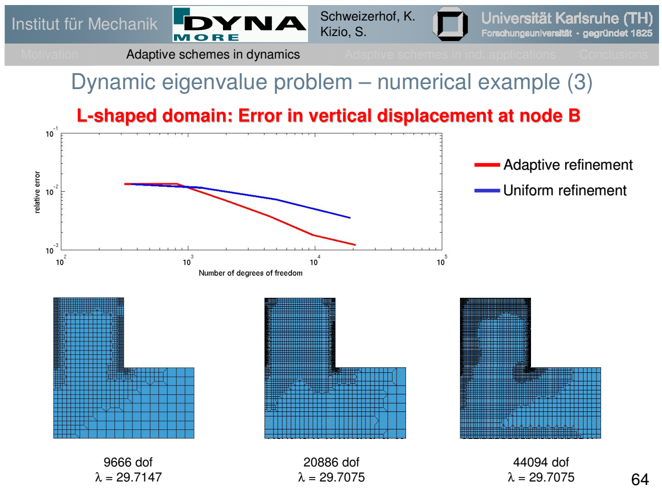
$$|E(\boldsymbol{U},\boldsymbol{U}_h)| \approx a(\boldsymbol{e}_u,\boldsymbol{e}_u)^{\frac{1}{2}} \cdot a(\boldsymbol{e}_z,\boldsymbol{e}_z)^{\frac{1}{2}} = ||\boldsymbol{e}_u||_a \cdot ||\boldsymbol{e}_z||_a$$

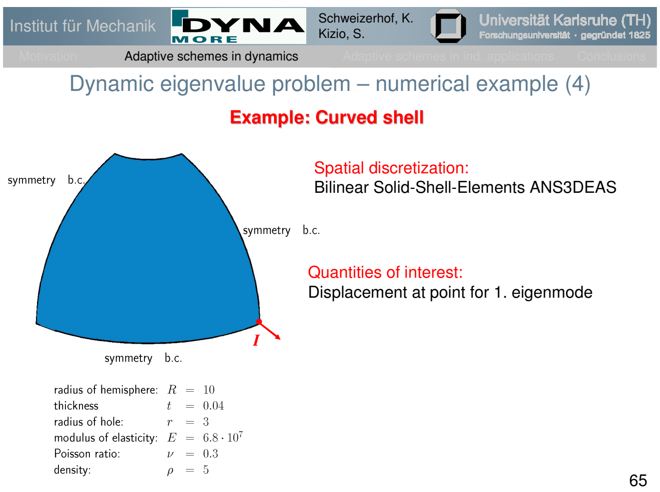
Here: Estimation of energynorm error of primal and dual problem with e.g. Zienkiewicz-Zhu estimator

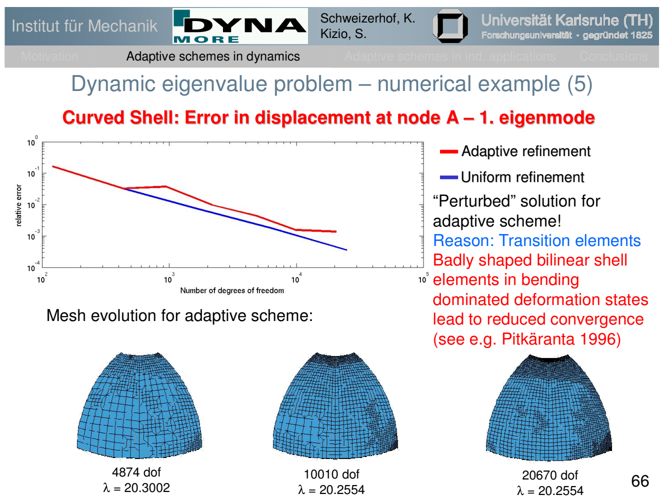

Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – adaptive scheme
Initial discretization
Definition of upper error bound ε_r
Solve eigenvalue problem
Adaptive mesh refinement
Solve dual problem
Error estimation $E(U, U_z)$
No
$|E(U, U_z)| < \varepsilon_r$?
Yes
End
Refinement scheme:
Hierarchical mesh refinement with transition elements
Numerical effort:
Several computations of eigenvalue problem and dual problem necessary
61


Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – numerical example
Example: Plate vibration; L-shaped domain
fully clamped
B
A
1
1
1
1
y
x
z
Material data
thickness $t = 0.1$
modulus of elasticity: $E = 3 \cdot 10^7$
Poisson ratio: $\nu = 0.3$
density: $\rho = 2.5$
Spatial discretization:
Bilinear Solid-Shell-Elements ANS3DEAS
Quantities of interest:
Displacements in z-direction at points A and B for 1. eigenmode
e.g. for control of max. displacement due to design requirements
62


Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – numerical example (2)
L-shaped domain: Error in vertical displacement at node A
relative error
Number of degrees of freedom
Adaptive refinement
Uniform refinement
9042 dof
λ = 29.7164
19476 dof
λ = 29.7079
41418 dof
λ = 29.7079
63


Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – numerical example (3)
L-shaped domain: Error in vertical displacement at node B
relative error
Number of degrees of freedom
Adaptive refinement
Uniform refinement
9666 dof
λ = 29.7147
20886 dof
λ = 29.7075
44094 dof
λ = 29.7075
64


Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – numerical example (4)
Example: Curved shell
symmetry b.c.
symmetry b.c.
symmetry b.c.
I
Spatial discretization:
Bilinear Solid-Shell-Elements ANS3DEAS
Quantities of interest:
Displacement at point for 1. eigenmode
radius of hemisphere: $R = 10$
thickness $t = 0.04$
radius of hole: $r = 3$
modulus of elasticity: $E = 6.8 \cdot 10^7$
Poisson ratio: $\nu = 0.3$
density: $\rho = 5$
65


Institut für Mechanik
DYNA MORE
Schweizerhof, K.
Kizio, S.
Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825
Motivation
Adaptive schemes in dynamics
Adaptive schemes in ind. applications
Conclusions
Dynamic eigenvalue problem – numerical example (5)
Curved Shell: Error in displacement at node A – 1. eigenmode
relative error
Number of degrees of freedom
Adaptive refinement
Uniform refinement
"Perturbed" solution for adaptive scheme!
Reason: Transition elements
Badly shaped bilinear shell elements in bending dominated deformation states lead to reduced convergence (see e.g. Pitkäranta 1996)
Mesh evolution for adaptive scheme:
4874 dof
λ = 20.3002
10010 dof
λ = 20.2554
20670 dof
λ = 20.2554
66

Dynamic eigenvalue problem - Conclusions

Error indicator

- Focus on single eigenfrequencies
- Standard global error indicators can be used

Mesh adaptation

- Hierarchical mesh adaptation with bilinear elements superior to uniform refinement for plane geometry (as expected)
- Curved shells with bilinear elements transition elements may yield perturbed solution in bending dominated case
  ➔ lower eigenmodes are usually bending dominated
  (effect is considerably reduced with biquadratic elements)
- Problems: adaptive refinement with transition elements for highly imperfection sensitive problems
  ➔ e.g. for static buckling of shells perturbed eigenmodes
  ➔ BUT less sensitive for dynamic postbuckling analysis

Adaptive finite element schemes and practical applications

Adaptive schemes are highly sophisticated but often suffer from limitations as:

- Only geometries with reduced complexity are considered
- Fairly uniform constitutive models and similar kinematic assumptions for whole domain are taken into account

But **practical (industrial) applications** consist of:

- Complex geometries such as e.g. shell intersections and many engineering model assumptions
- Combination of different element types
  - Assumed/enhanced strains
  - Selective/underintegration (➔ hourglassing)
- Combination of rigid bodies and elastic parts
- Constraints, Spotwelds, Joints etc.

For **practical problems**: Mathematically strict treatise of the problem yields very high effort ➔ Simplifications necessary

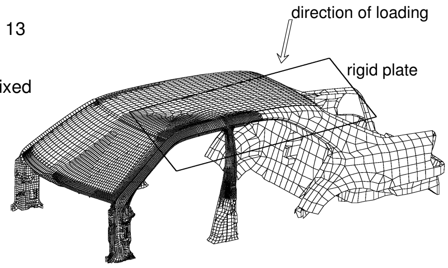
Industrial adaptive computations in automotive industry

Roof Crush Problem using adaptivity

Model data:

- 27712 shell elements
- 16115 nodes
- self contact type 13
- rigid wall contact
- lower boundary fixed
- applied velocity

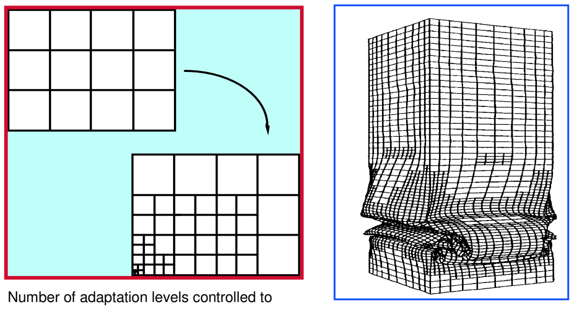
Fairly complex problems with multiple shell intersections



Examples of adaptive computations in automotive industry

Mesh refinement scheme in LS-DYNA

Hierarchical mesh - One side two-rule – hanging node strategy



Number of adaptation levels controlled to keep certain time step size – explicit time integration

Adaptive computations in automotive industry

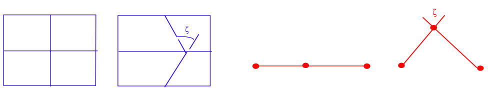
Mesh refinement scheme e.g. in commercial program LS-DYNA

Goal: *Efficient, element independent, must work also for shell intersections*

Refinement indicators: ➔ **dedicated to detect large deformations**

- **Angle based indicators**
  ⇨ **Check deformation between elements**

in plane **out-of-plane**



undeformed deformed undeformed deformed

⇨ **Total angle change - for duration of calculation**

⇨ **Incremental angle change - between adaptive steps**

Remark: Tolerance values from experience / trial runs

Adaptive computations in automotive industry

Mesh refinement scheme e.g. in commercial program LS-DYNA

Refinement indicators (cont.)

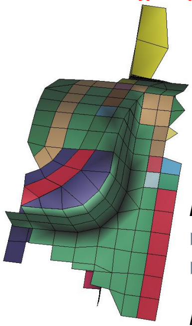
- **Thickness**
  ⇨ Refine based on thickness changes
- **Impending contact** - **action in advance**
  ⇨ Refine before contact occurs
  ⇨ One pass adaptivity can be used
  ⇨ May create more elements than necessary.
- **Interpenetration in contact**
  ⇨ Small geometric features on one side of a contact interface contacting "large" elements on the other.
  - Two pass adaptivity is necessary

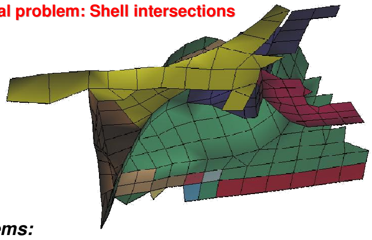
Remark: Tolerance values from experience / trial runs

- **no patch information necessary (or available) – locality important for parallel execution**

Adaptive computations in automotive industry

Typical practical problem: Shell intersections





Problems:

- Shell theory not valid throughout whole domain
- Error estimation: e.g. only in parts with pure shell theory *(Schweizerhof/Riccius 1995)*

But: Mesh adaptation in the vicinity of intersection?

Use simple indicators !

73

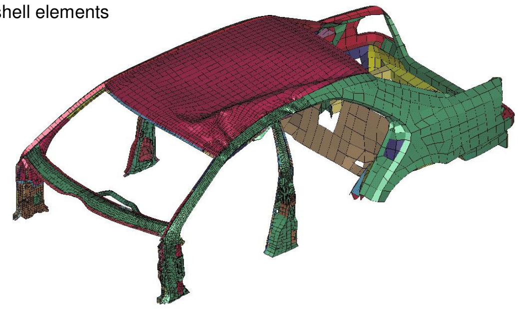
Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry with LSDYNA

Roof Crush Problem using adaptivity

Run without adaptive refinement:

- 27712 shell elements



74

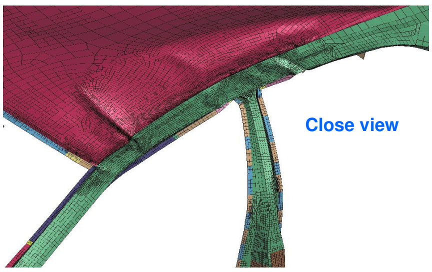
Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry with LSDYNA

Roof Crush Problem using adaptivity

Refinement level 3:

- 58926 shell elements



Close view

75

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Side aspects of adaptivity with commercial program - explicit time integration

Memory

- Memory allocation and disk space usage must be determined dynamically as the problem size grows

Contact

- Automatic contact definitions are required between parts which are adapted.
- Thickness offsets for shells are mandatory
- Adaptive constraints (due to hanging nodes) must be handled in contact searching

Mass scaling

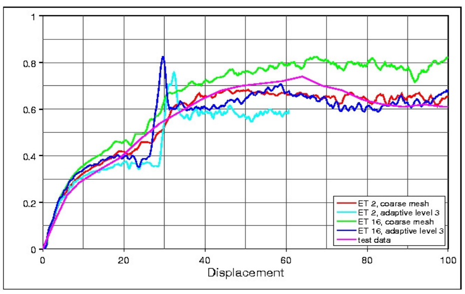
- Adaptivity leads to subdivision of elements
  ➔ Courant – F.-L. –conditions yields small timestep size (stability)
- Efficiency: Keep timestep size constant by mass scaling

76

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry

Roof Crush: Comparison with experimental data - **high** mass scaling



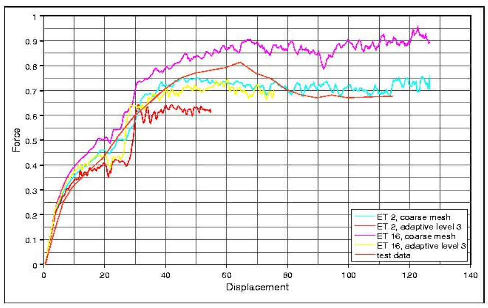
Artificial spike in response curve

77

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry

Roof Crush: Comparison with experimental data - **reduced** mass scaling



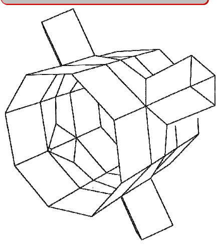
Artificial spike in response curve **removed**

78

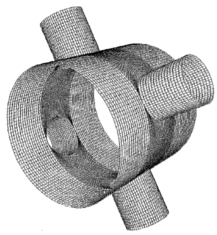
Adaptive computations in automotive industry

Desirable tool: Adaptive remeshing with/on CAD data

Coarse initial mesh



Refined mesh



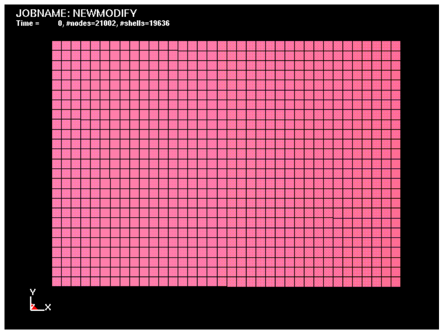
Problem:

Geometry for nonlinear problems with large deformations not known

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry – with LS-DYNA

Sheet metal forming problem



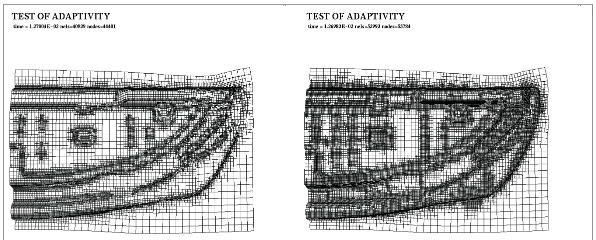
Refinement based on indicators for large deformation problems

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry with LS-DYNA

Sheet metal forming problem

Effect of ref.-indicators: ANGLE CHANGE vs. IMPENDING CONTACT



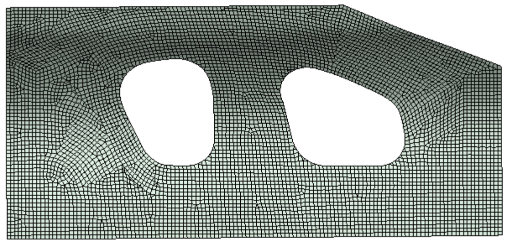
ANGLE CHANGE indicator | IMPENDING CONTACT indicator

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes in industrial applications | Conclusions

Adaptive computations in automotive industry with LS-DYNA

Sheet metal forming problem

Combined use of refinement indicators
ANGLE CHANGE **and** IMPENDING CONTACT



➢Simple transfer of data due to hierarchical meshing and refinement only
Second pass analysis with final refined mesh *(for verification because of contact)*

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Using adaptivity for model reduction – from flexible to rigid body analysis

Goal: For many engineering problems efficient e.g rigid body models are mandatory
- for long time analyses
- for design studies of fairly rigid machine parts with "minor" modifications

Current situation
- Rigid body models are created simply on engineering assumptions

Improvement
- Verify resp. create rigid body models by adaptivity
- Monitoring deformation indicators

Examples
- Simple transient problem
- Verifying/creation of rigid bodies for efficient demolition simulations in case of demolition using explosives

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825
Motivation | Adaptive schemes in dynamics | Adaptive schemes for model reduction | Conclusions

Using adaptivity for model reduction – from flexible to rigid body analysis

General procedure

Start computation with flexible structure
↓
Detect parts that can be simplified to rigid behavior
↓
Compute rigid body data for new rigid parts, using FE-discretization
↓
Transform rigid parts back to flexible parts if desired

- **Coupling between rigid and flexible parts:** Transformation to minimal coordinates
- **Time Integration method:** implicit midpoint rule
  Specific variation Energy Momentum Method [Simo et al, 1992]
- **Solution Method for nonlinear equations:** Implicit Newton - Raphson

Using adaptivity for model reduction - from flexible to rigid body analysis

Criterion to switch status from flexible to rigid

Check at each time step for *every* flexible element:
Normalized strain rate

$$e^e = \frac{\sqrt{\int (\mathbf{E}_{10}-\mathbf{E}_{00}) : (\mathbf{E}_{10}-\mathbf{E}_{00})\ dV_0^e}}{\Delta t\ V_0^e} \le tol$$

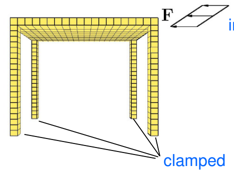
$\mathbf{E}$ Green-Lagrange strain tensor
V_0^e Element volume (reference)
Δt time step size
00/10 beginning / end of time step

- strain rate based, geometrical and objective criterion.
- directly correlated to discretization.
- efficient ($\mathbf{E}$ also needed for stiffness matrix)

85

Using adaptivity for model reduction - from flexible to rigid body analysis

Frame Structure



Geometry:
Plate: 400 x 200 x 20 [cm]
Column: 20 x 20 x 300 [cm]

Material: Youngs-Modulus E: 2.1 $10^5 \frac{N}{mm^2}$
Poisson ratio ν : 0.2
mass density: 8 $\frac{g}{cm^3}$
Num. damping [Armero,Petöcz 1998]

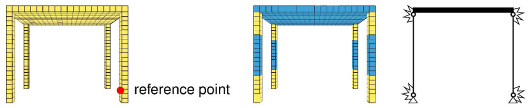
Loading: $0.0 \le t \le 0.05s$ Forces $\sum \mathbf{F} = 11000N$
Time step size: $0.05\,s$ (Energy-Momentum-Scheme)

Rigid body detection
Check criterion in every time step and mark violating elements
Check allocation rigid - flexible in time interval: $\Delta t = 0.5\,s$

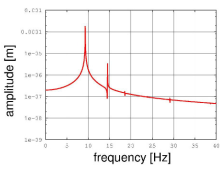
86

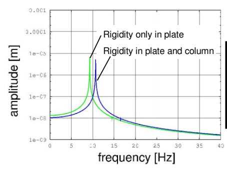
Frame Structure: Results

$T = 0.05s$ $T = 2.00s$



Amplitude-Frequency Spectra for horizontal displacement of reference point





Further eng. action:
Joints must be more flexible

87

Using adaptivity for model reduction - from flexible to rigid body analysis

Lime works Borna, Germany

Application to building demolition by explosives

Silo in Weimar, Germany
Videos by courtesy of Dr.-Ing. Rainer Melzer
Planungsbüro für Abbruchsprengen

Construction data

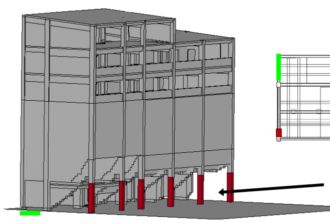
- **Height 25.2 m length 36.0 m width 12.0 m**
- **reinforced concrete construction with concrete and masonry walls**
- **Main part silo installations**
- **Mass of building is approximately 3630 t**

Demolition companies:
-Thüringer Sprenggesellschaft
-Dr.Ing. R. Melzer

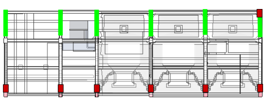
88

Borna - Blasting strategy

Structure should topple in direction of weak axis



Ground plan



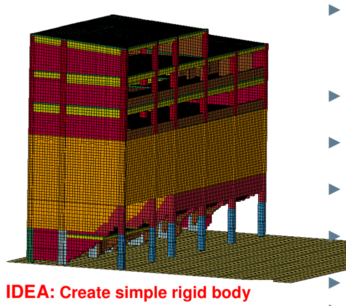
Wedges in the front part are demolished conventionally

Removed columns   Reinforcements are cutted

89

Using adaptivity for model reduction - from flexible to rigid body analysis

Computational Model - Discretization



IDEA: Create simple rigid body model via flexible 3D FE model for fast analysis using "adaptivity"

- **Building discretized via 75570 hexahedral elements (reduced integrated, hourglass stabilization)**
- **Ground plate discretized via 1330 shell elements**
- **Elastic-plastic material model used**
- **Failure modeled via element erosion (*criterion: plastic strains*)**
- **Contact via segment-to-segment penalty formulation**
- **FE-Code LS-DYNA**
- **Blast process modeled by removing elements**

90

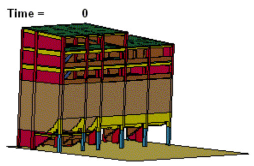
Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes for model reduction | Conclusions

Using adaptivity for model reduction - from flexible to rigid body analysis

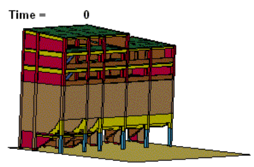
Results – numerical analysis

(Controlled demolition)



Model A
(no fracture in silo assumed)
analyzed with flexible parts

Visual comparison shows good agreement



Model B
(**no fracture** in silo/columns assumed)
analyzed with flexible parts

Visual comparison shows good agreement

91

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes for model reduction | Conclusions

Using adaptivity for model reduction - from flexible to rigid body analysis

Validation – video source – Model B

Common overlay of numerical analysis results and video detail screen

Result:

Location of fracture adequate /

Kinematics are similar

Problem *(of validation)*:
Only a single video available

For rendering process location and view angle not known

92

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes for model reduction | Conclusions

Using adaptivity for model reduction – from flexible to rigid body analysis

Application to demolition by explosives:

Nice idea - BUT:

- Application to 3D structure not straightforward **– not automatic**
- For hinge evolution fairly detailed model necessary
- Hinge lines develop – resolution/discretization must capture form
- Fracture must be captured somehow
- Contact evolution must be captured – rigid body must contain appropriate joints/hinges
- Many more difficulties …..

Project in progress – FOR500 research group involving Universities Bochum-Dresden-Karlsruhe

93

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Conclusions

- **Goal oriented error estimators** based on dual weighted residual very powerful method for mesh adaptation in structural dynamics
- Applications and problems discussed
  - computational effort, transfer of data etc.
- Adaptive schemes are usually designed and tested on rather simple "academical" problems, e.g. simple geometries, uniform problems
- Simplifications are necessary for practical – large scale - applications of adaptive schemes *(due to computational effort needed)*
- For **different problem classes simplifications** are possible, e.g.:
  - Vibration dominated problems
  - Large deformation problems involving contact

  Consequences: Error estimation /refinement indication is limited, **but shows fairly good results for eng. purposes**

94

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Motivation | Adaptive schemes in dynamics | Adaptive schemes in ind. applications | Conclusions

Conclusions/Outlook

- Practical problems – however - usually consist of
  - very complex geometries, shell intersections,
  - combinations of elements – beams, shells, solids
  - constraints with arbitrary conditions, point connections etc.

  Consequences:
  - In simulation programs with adaptive algorithms many complex algorithmic details are needed beyond adaptivity
  - Simple (though rather limited) algorithms are necessary for robustness
- Adaptive schemes/error estimation further provide hints for modeling and discretization
  - Many useful hints for model limits can be obtained
  - Application for model reduction – creation of engineering models
  - BUT: Automatic creation via application of adaptive algorithm to find simple models not straightforward *(if at all possible)*

95

Institut für Mechanik | DYNA MORE | Schweizerhof, K. Kizio, S. | Universität Karlsruhe (TH) Forschungsuniversität · gegründet 1825

Thank you for your attention !

96